A bit more on synchronous rewriting

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Open problems in machine translation

Machine translation of speech and informal text

Spanish	E- ella su ma- el marido de ella es de aquí ¿verdad?
English	H- her husb- her husband is from here, right?
MT	E-ma-she her husband she is here right?

Spanish	yo la kiero ver pork me encanta Yonghwa jijiji
English	I want to see it because I love Yonghwa hehehe
MT	I love pork kiero see Yonghwa jijiji

Some open problems in machine translation

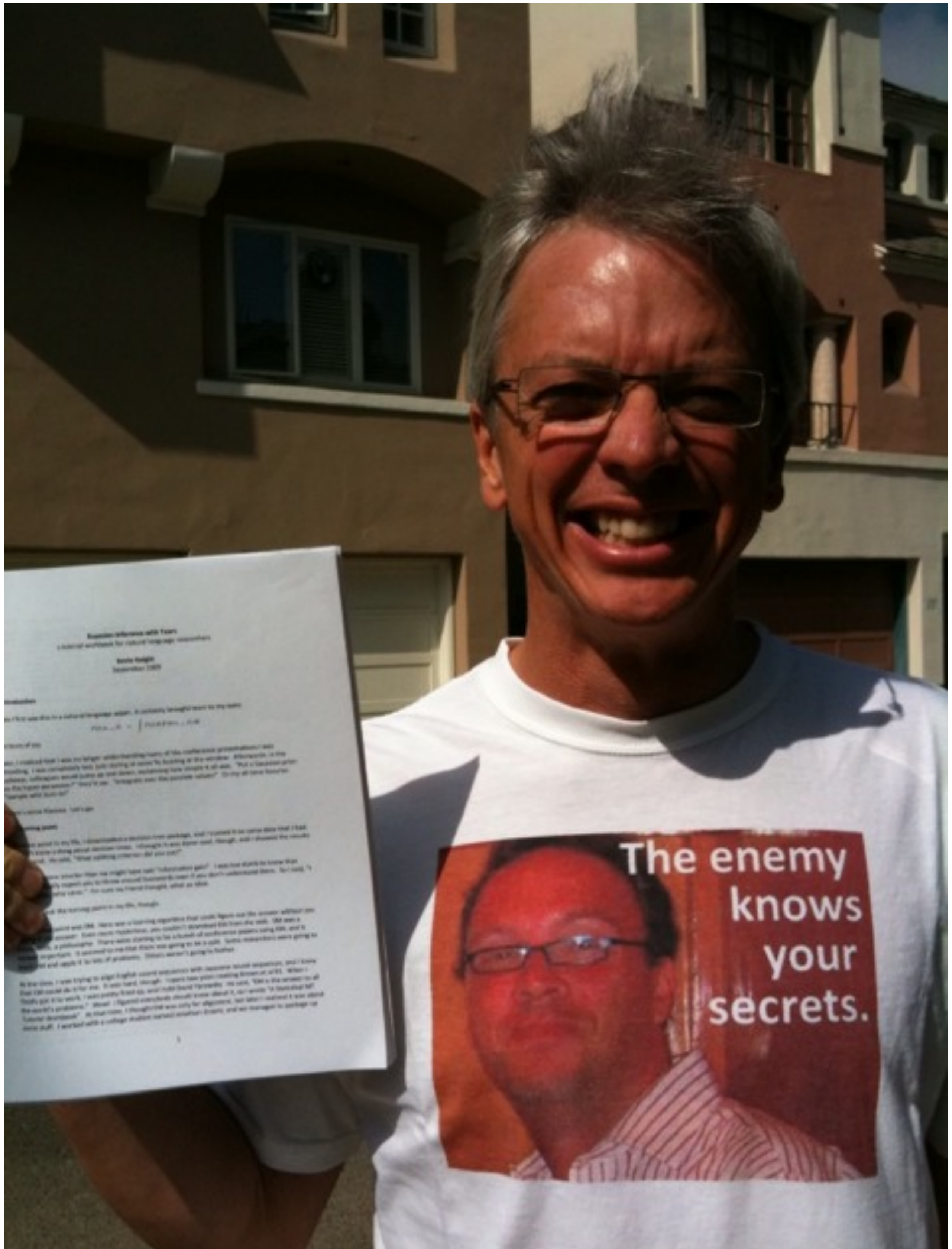
Machine translation of complex **syntactic**, **semantic**, and morphological phenomena

German	Anna fehlt ihrem Kater	
English	Anna's cat is missing her	(Jones et al. 2012)
MT	Anna is missing her cat	

Dutch	omdat ik Cecilia Henk de nijlpaarden zag helpen voeren
English	because I saw Cecilia help Henk feed the hippopotamuses
MT	because I saw the hippos help implement Cecilia Henk

English	the allocation of	resources	has completed
Russian	распределение	ресурсов	завершено
Gloss	NN+sg+nom+neut	NN+sg+gen+pl+masc	VERB+perf+pass+part+neut+sg

Machine Translation = Automata Theory + Probability + Linguistics



Kevin Knight

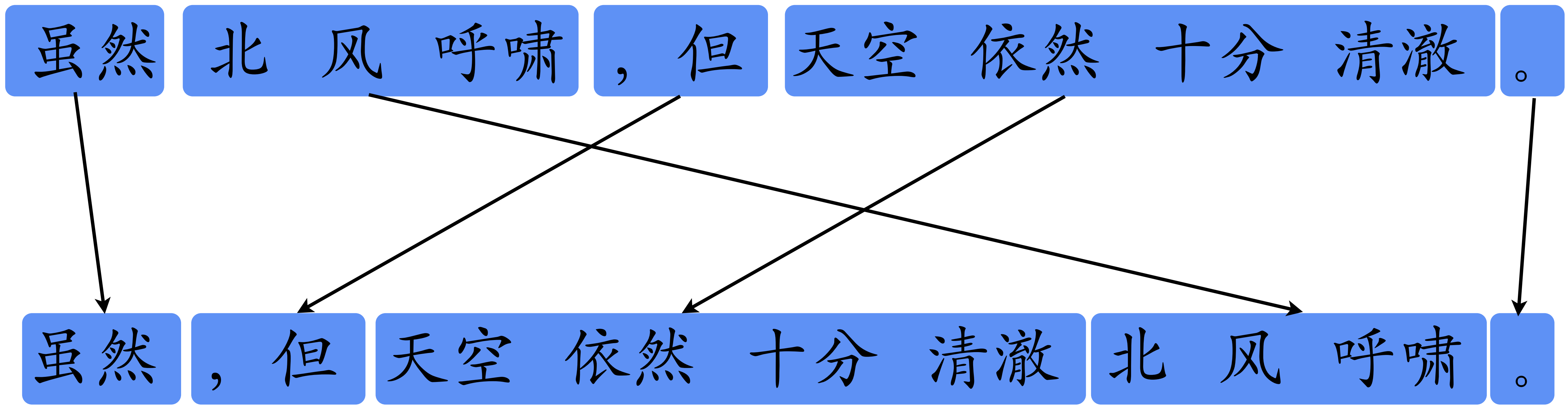
Phrase-based translation

虽然 北 风 呼啸 ， 但 天空 依然 十分 清澈 。

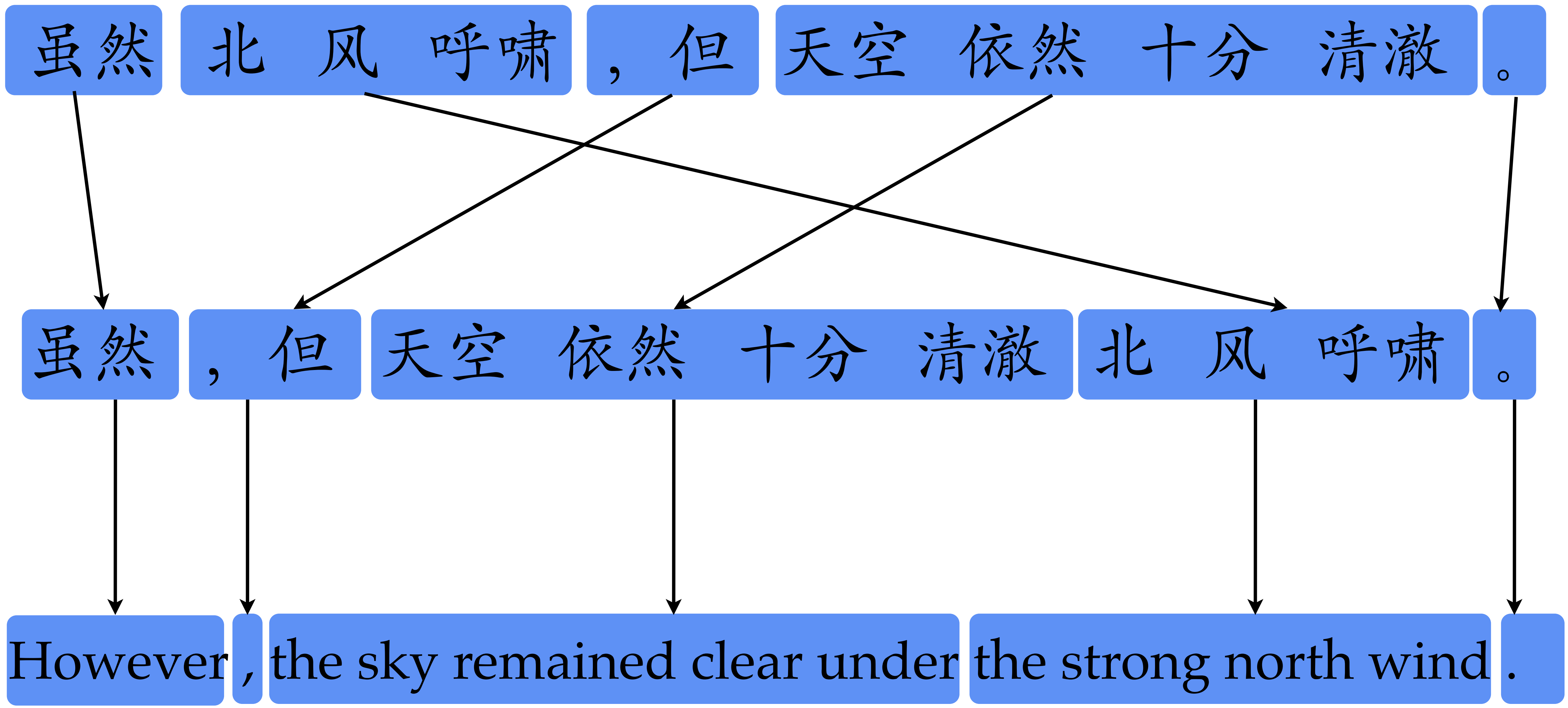
Phrase-based translation

虽然	北 风 呼啸	， 但	天空 依然 十分 清澈	。

Phrase-based translation



Phrase-based translation



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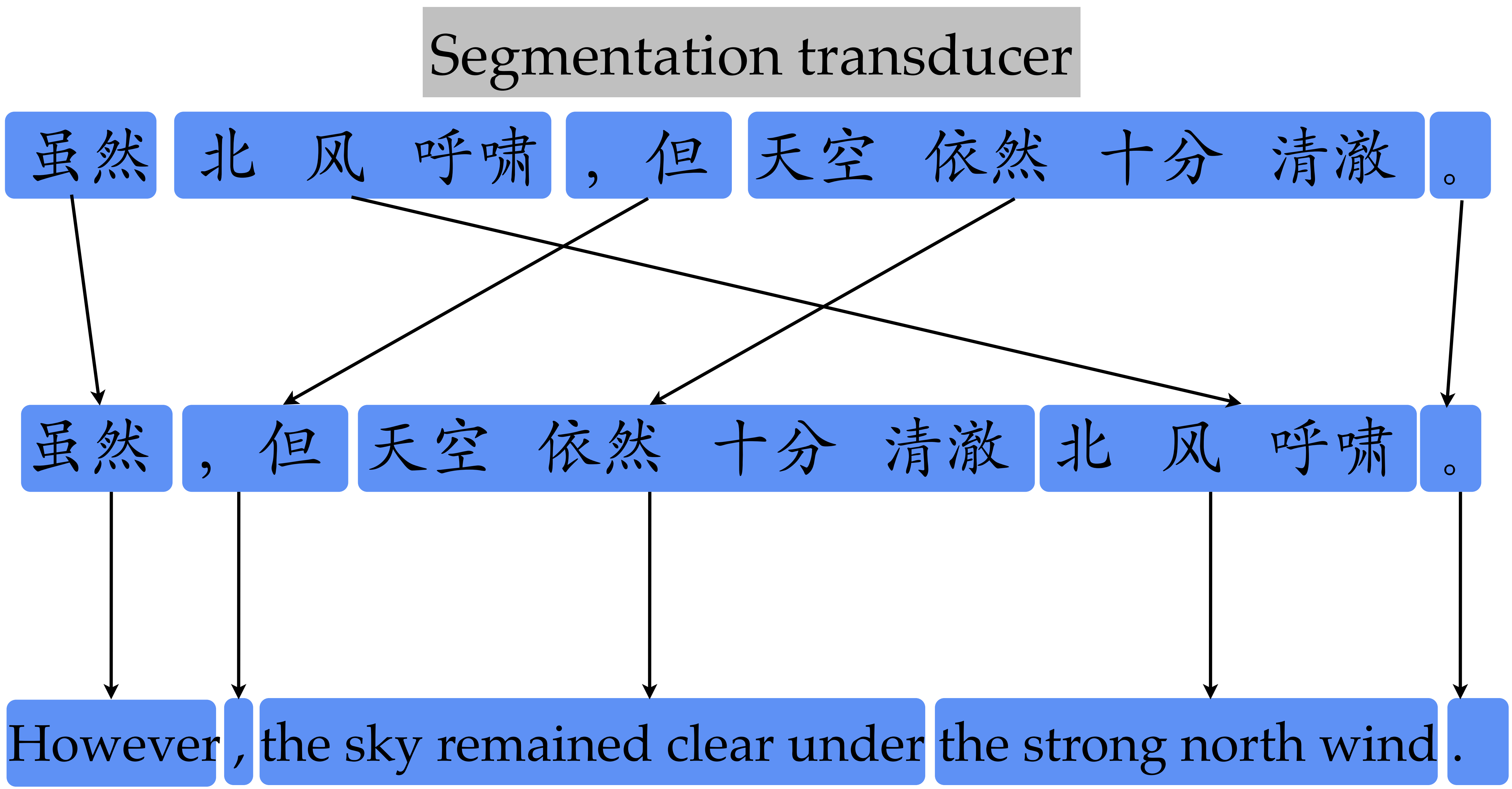

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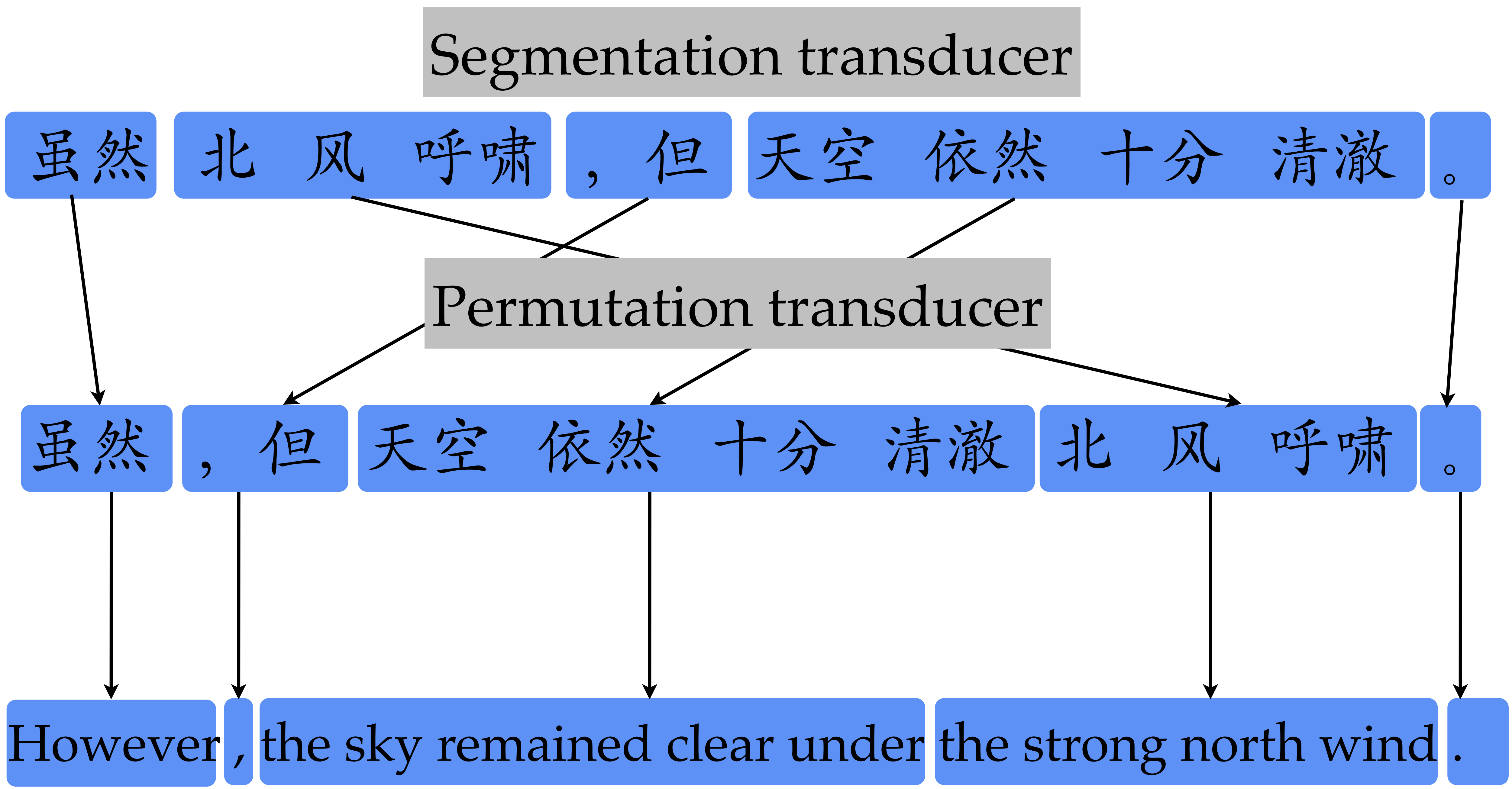

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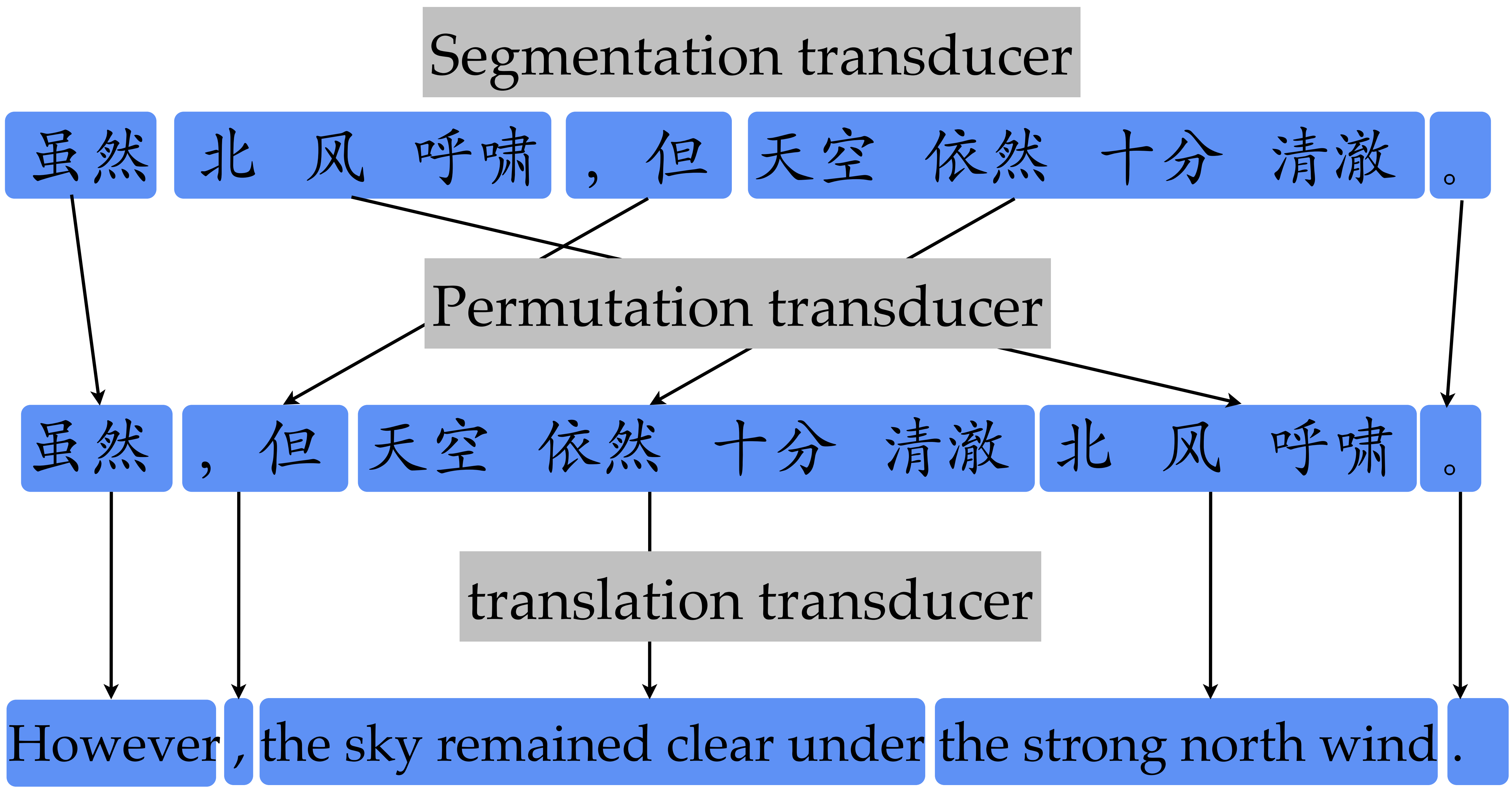
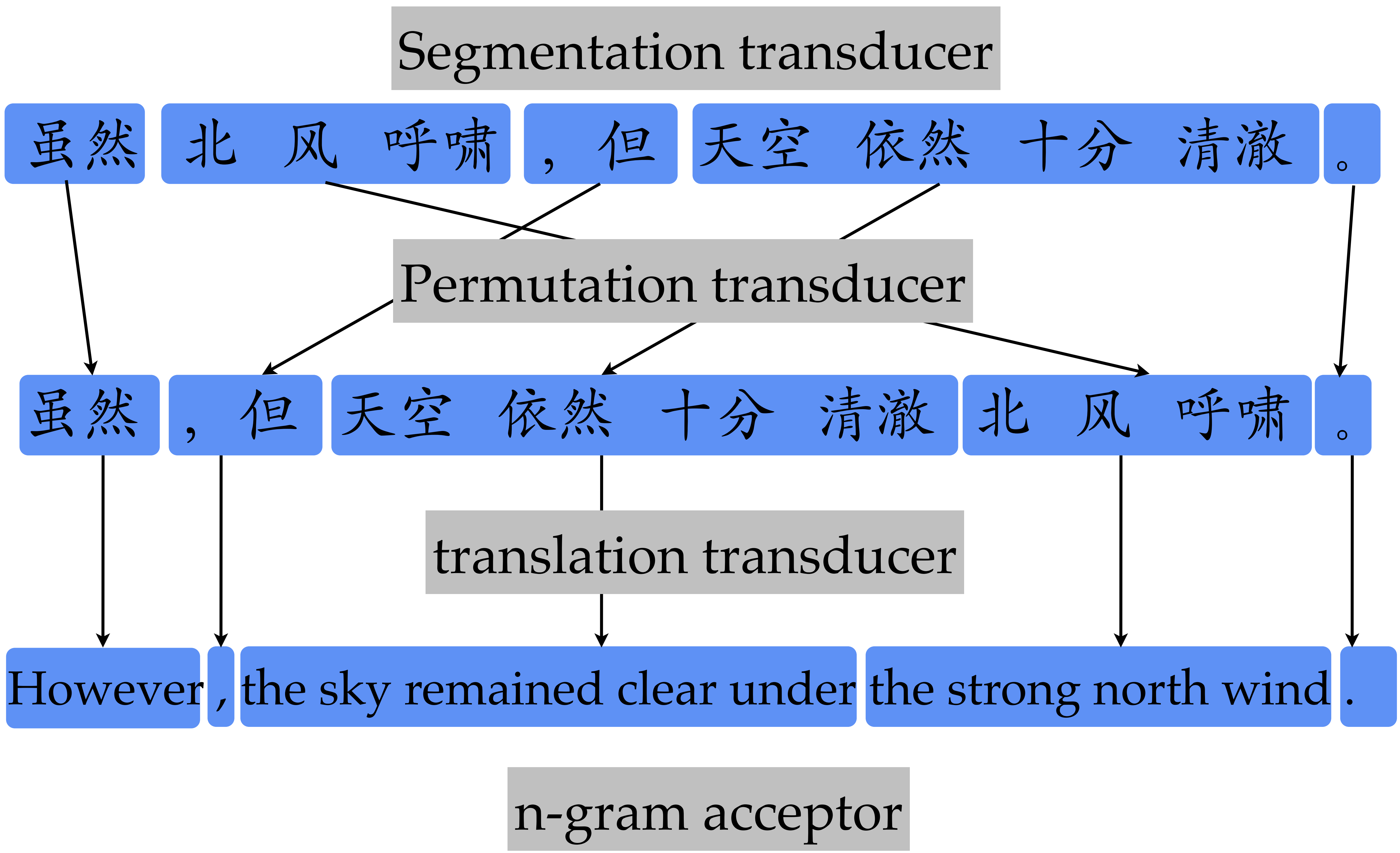
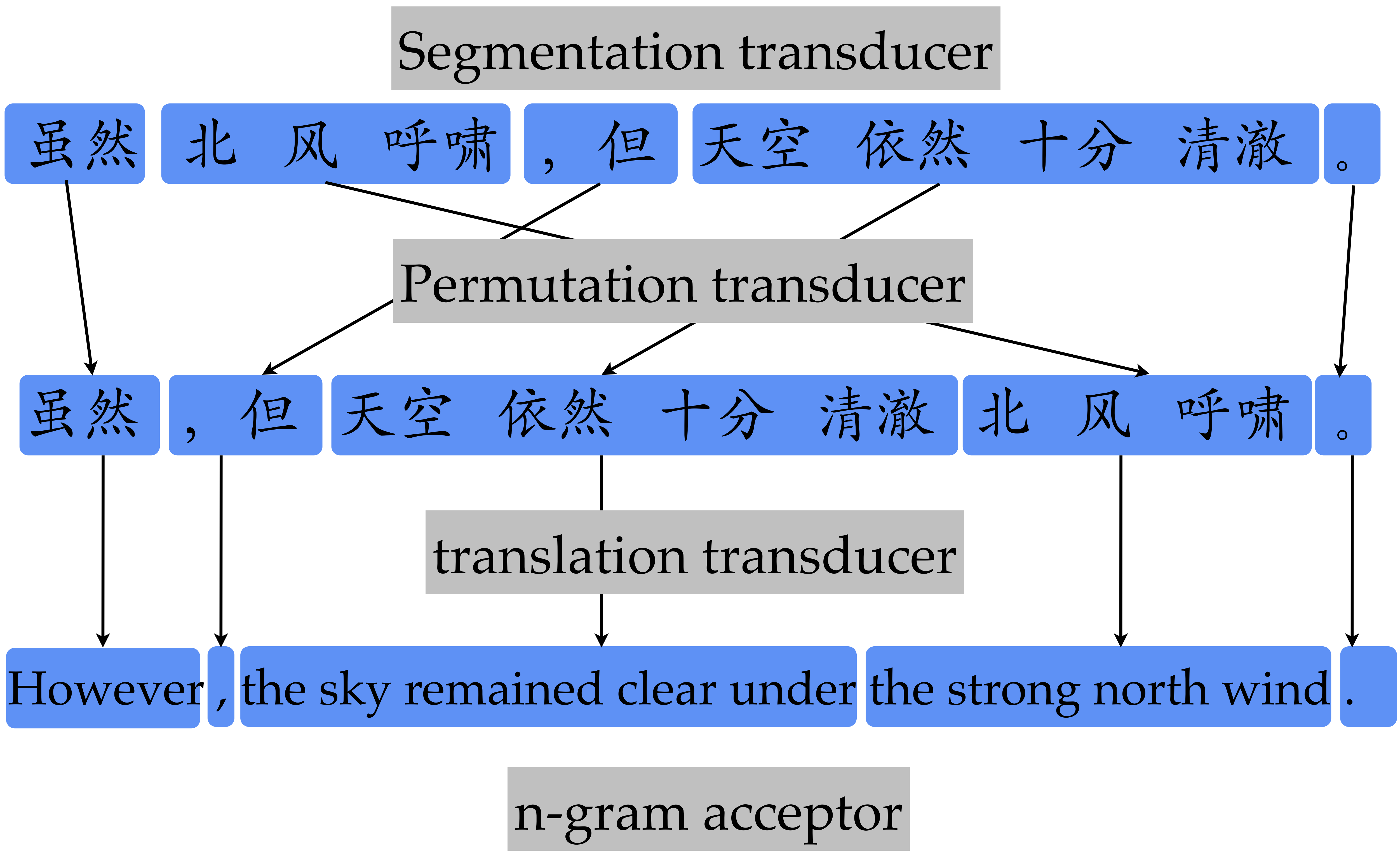


Phrase-based translation



Phrase-based translation

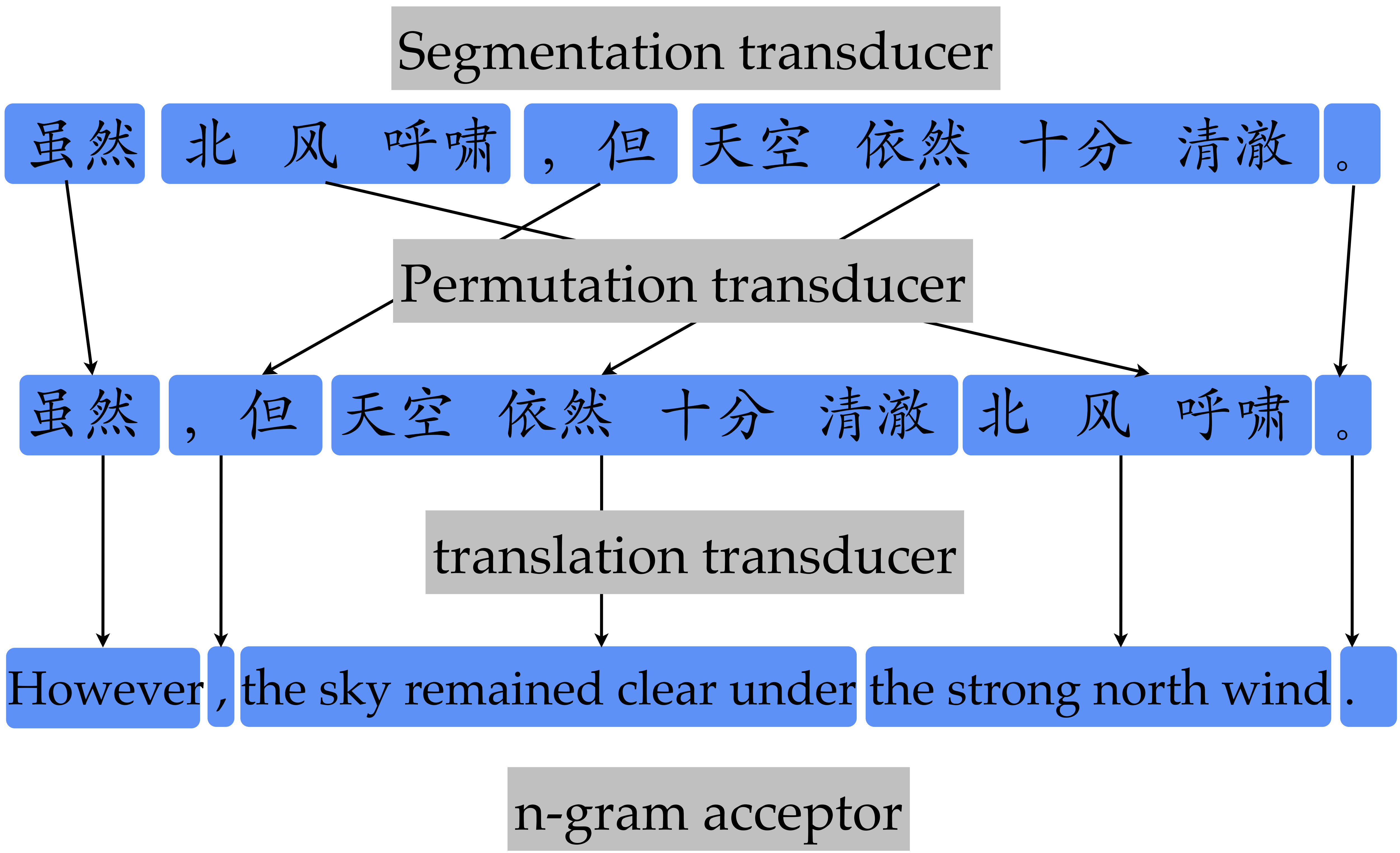
= weighted Finite-State Transduction



Phrase-based translation

= weighted Finite-State Transduction

$$I \circ S \circ P \circ T \circ L$$



Synchronous context-free grammar

AKA syntax-directed translation (Lewis & Stearns 1966; Aho and Ullman 1969)

$S \rightarrow NP\ VP$

$NP \rightarrow$ watashi wa

$NP \rightarrow$ hako wo

$VP \rightarrow NP\ V$

$V \rightarrow$ akemasu

Synchronous context-free grammar

AKA syntax-directed translation (Lewis & Stearns 1966; Aho and Ullman 1969)

S → NP VP	S → NP VP
NP → watashi wa	NP → I
NP → hako wo	NP → the box
VP → NP V	VP → V NP
V → akemasu	V → open

Synchronous context-free grammar

AKA syntax-directed transduction (Lewis & Stearns 1966; Aho and Ullman 1969)

$S \rightarrow NP_1\ VP_2\ ,\ NP_1\ VP_2$

$NP \rightarrow$ watashi wa , I

$NP \rightarrow$ hako wo , the box

$VP \rightarrow NP_1\ V_2\ ,\ V_2\ NP_1$

$V \rightarrow$ akemasu , open

Synchronous context-free grammar

$S \rightarrow NP_1\ VP_2\ ,\ NP_1\ VP_2$

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$NP \rightarrow$ hako wo , the box

$VP \rightarrow NP_1\ V_2\ ,\ V_2\ NP_1$

$V \rightarrow$ akemasu , open

Synchronous context-free grammar

S S

$S \rightarrow NP_1\ VP_2\ ,\ NP_1\ VP_2$

$NP \rightarrow$ watashi wa , I

$NP \rightarrow$ hako wo , the box

$VP \rightarrow NP_1\ V_2\ ,\ V_2\ NP_1$

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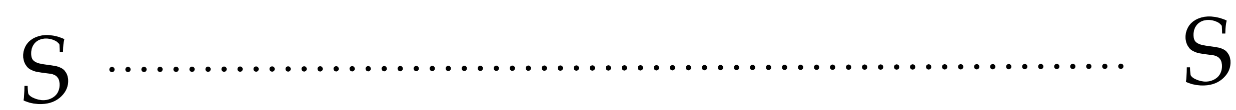
Synchronous context-free grammar



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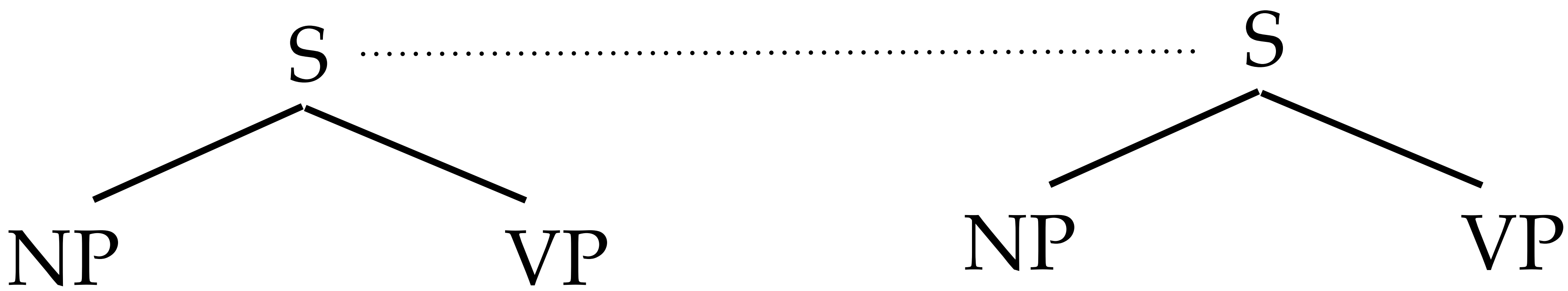

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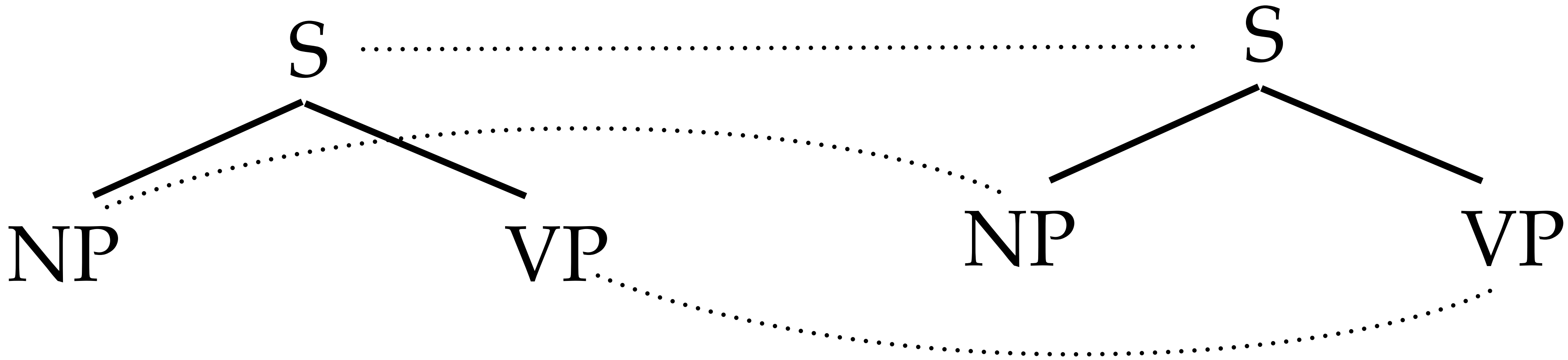

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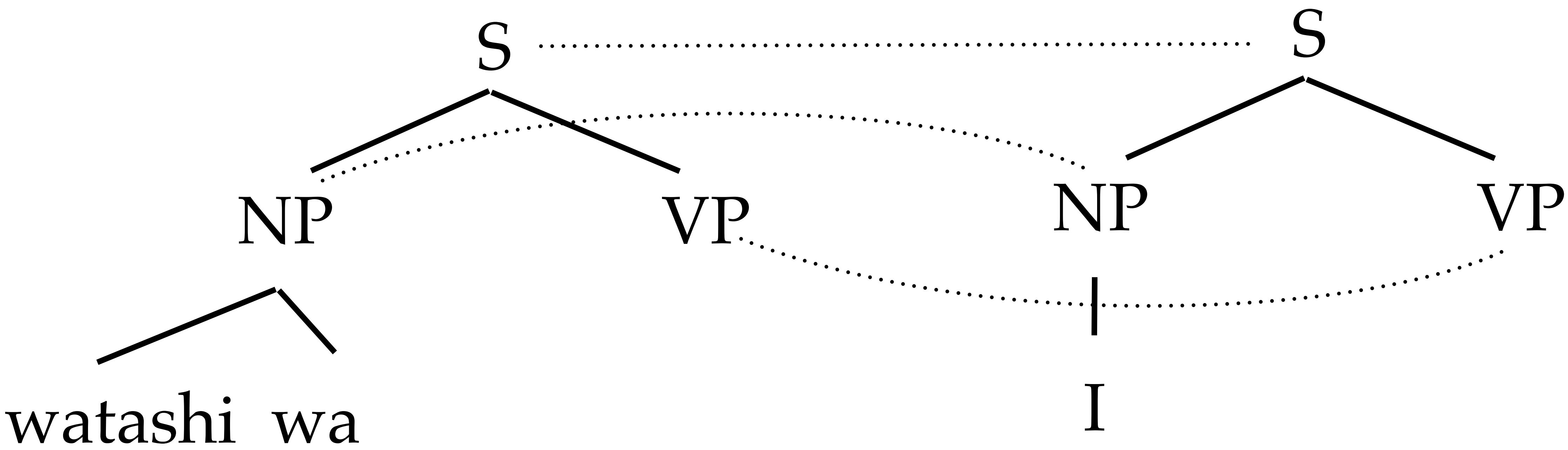

$S \rightarrow NP_1\ VP_2\ ,\ NP_1\ VP_2$

$NP \rightarrow$ watashi wa , I

$NP \rightarrow$ hako wo , the box

$VP \rightarrow NP_1\ V_2\ ,\ V_2\ NP_1$

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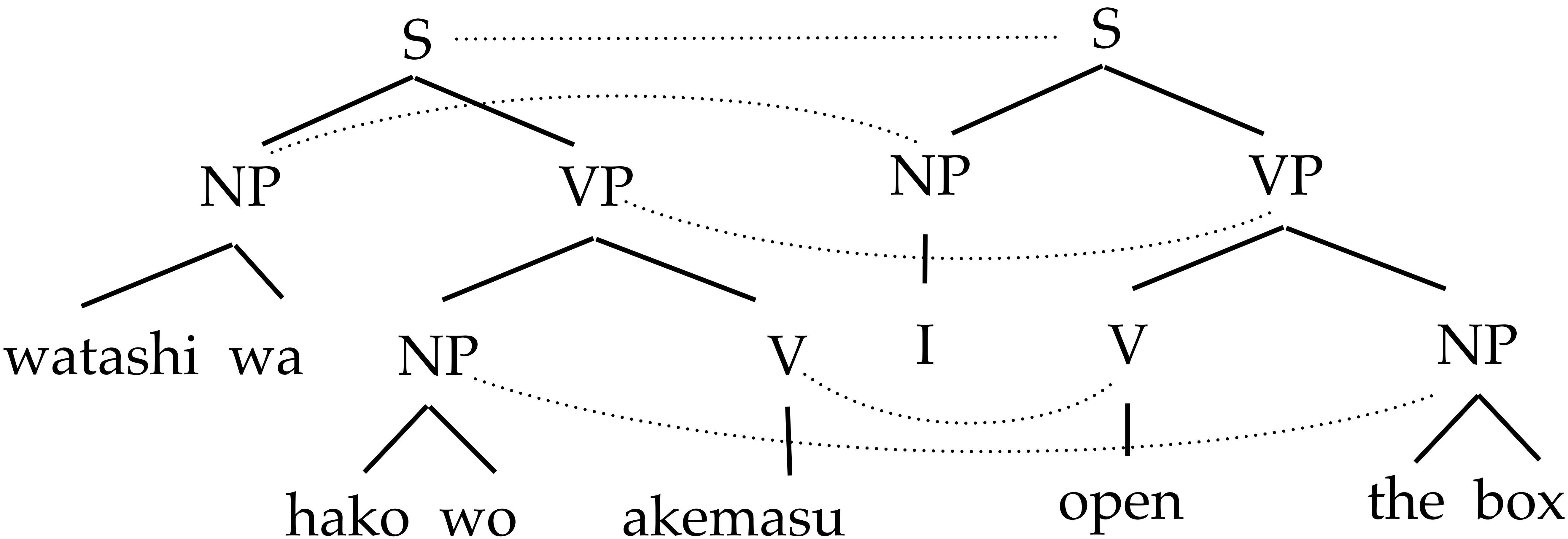
Synchronous context-free grammar



$$S \rightarrow NP_1\ VP_2\ ,\ NP_1\ VP_2$$

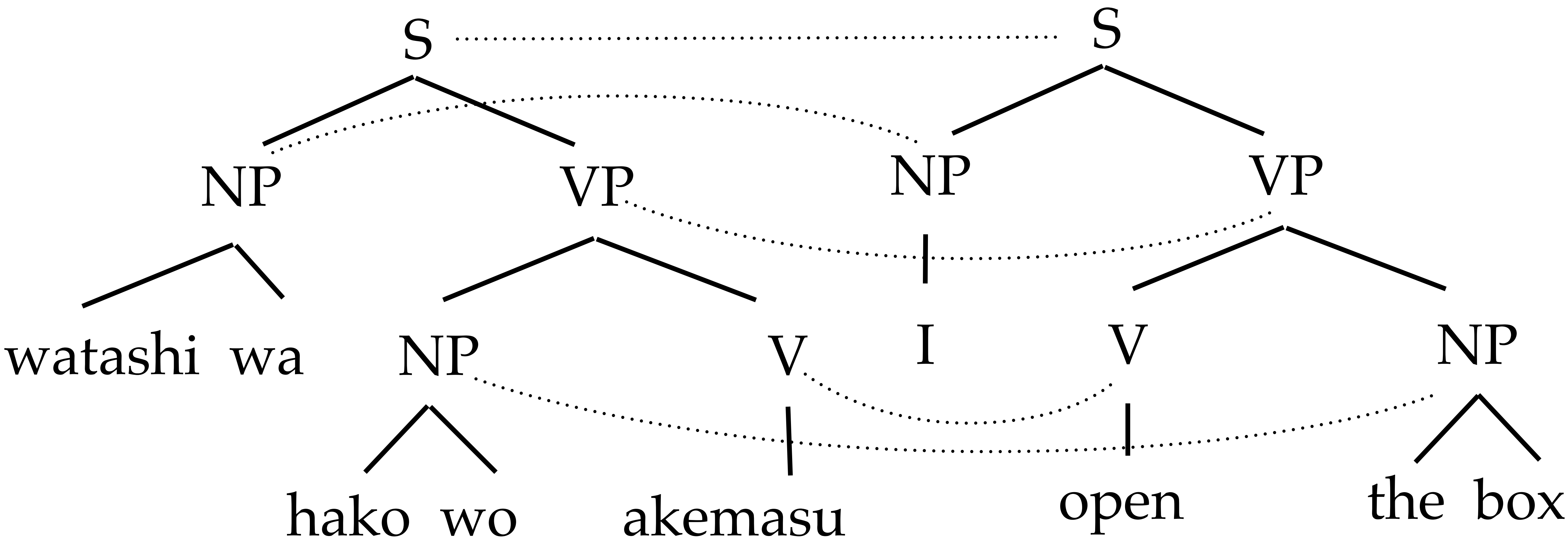
$$NP \rightarrow \text{watashi wa}\ ,\ \text{I}$$

$$NP \rightarrow \text{hako wo}\ ,\ \text{the box}$$

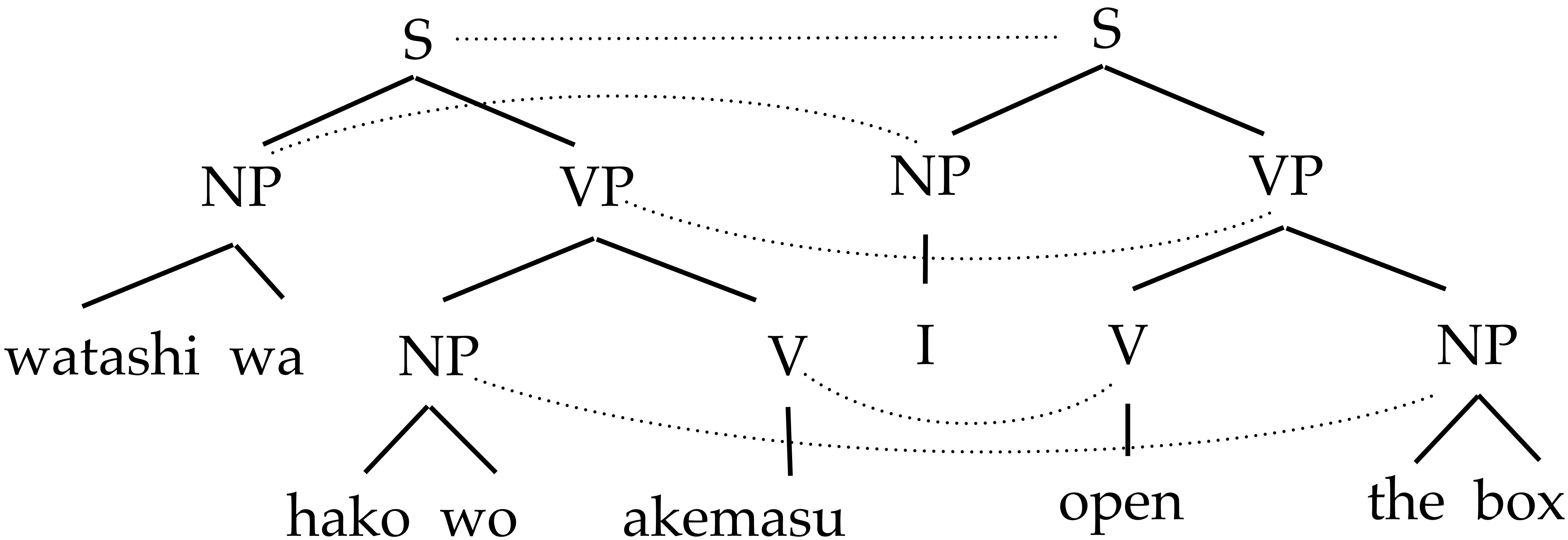
$$VP \rightarrow NP_1\ V_2\ ,\ V_2\ NP_1$$

$$V \rightarrow \text{akemasu}\ ,\ \text{open}$$

Synchronous context-free grammar

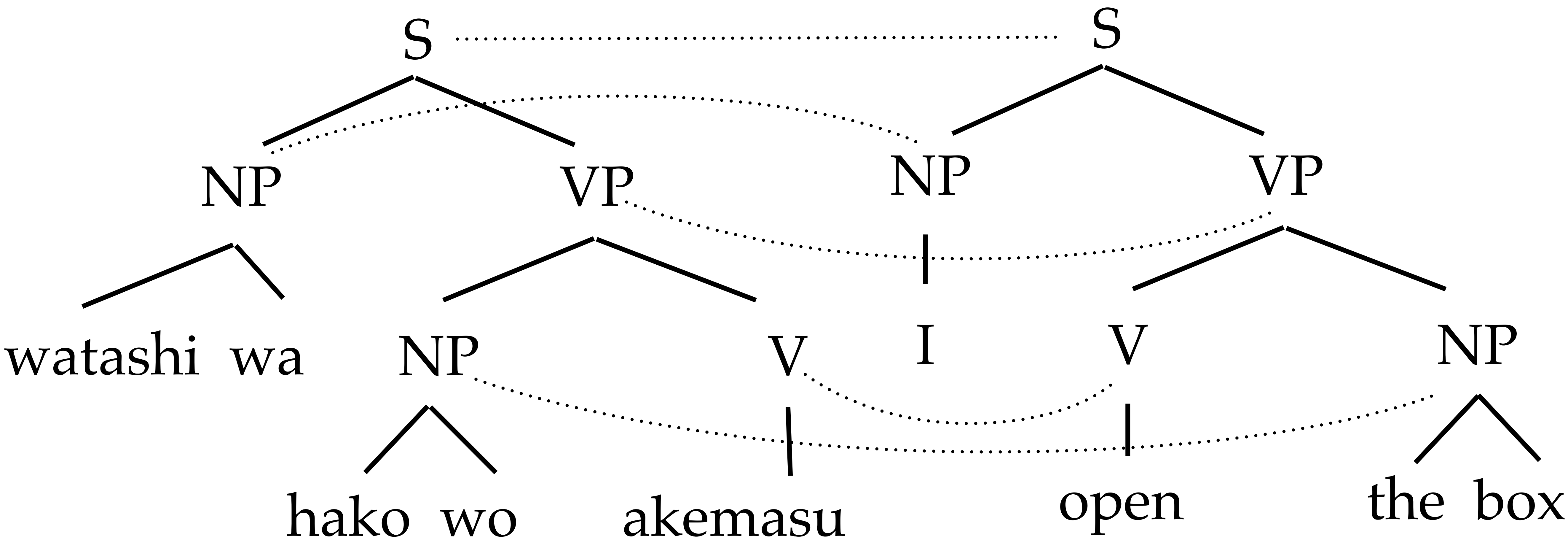


Synchronous context-free grammar



watashi wa hako wo akemasu

Synchronous context-free grammar



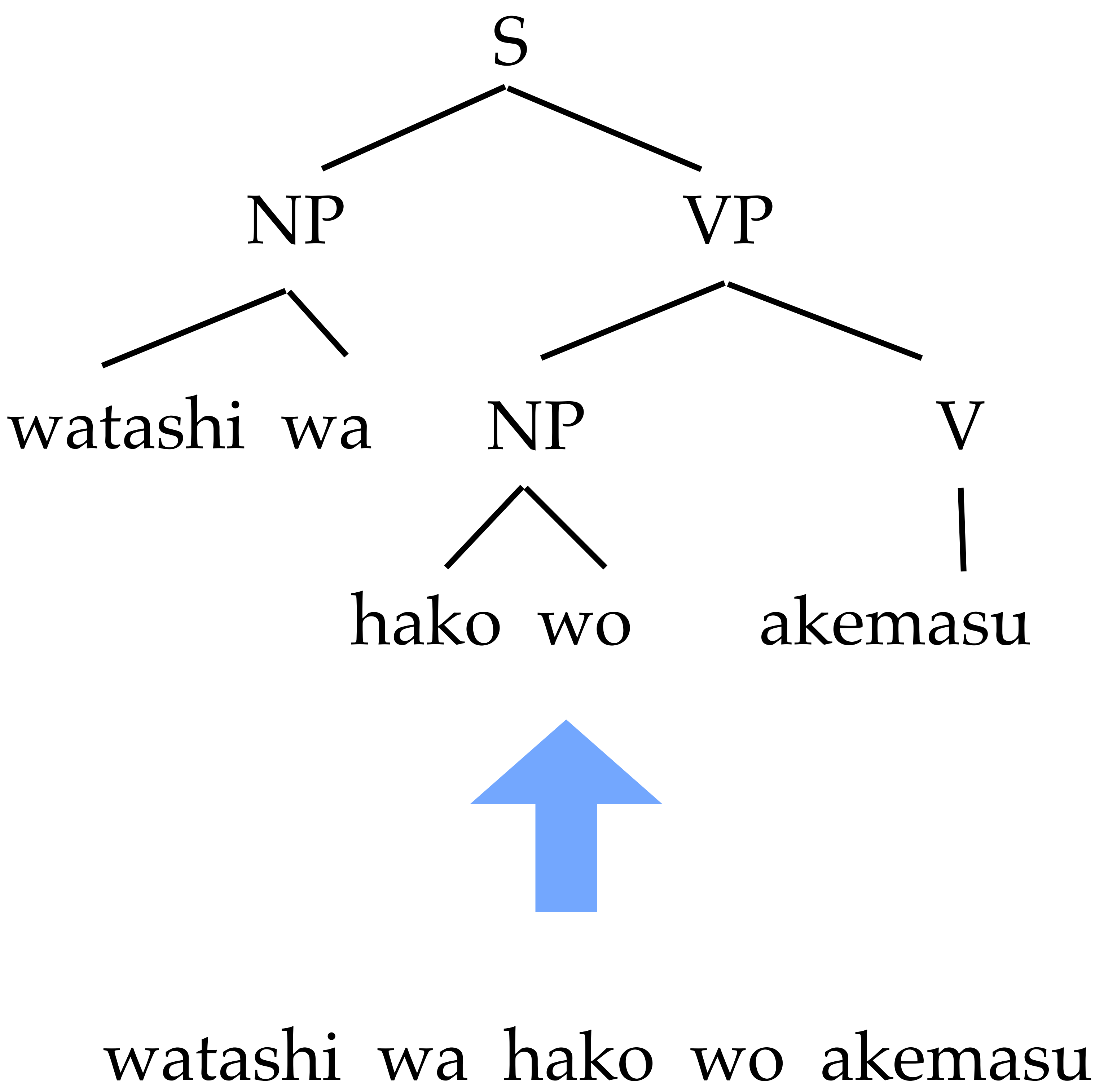
watashi wa hako wo akemasu

I open the box

Syntax-based translation

watashi wa hako wo akemasu

Syntax-based translation



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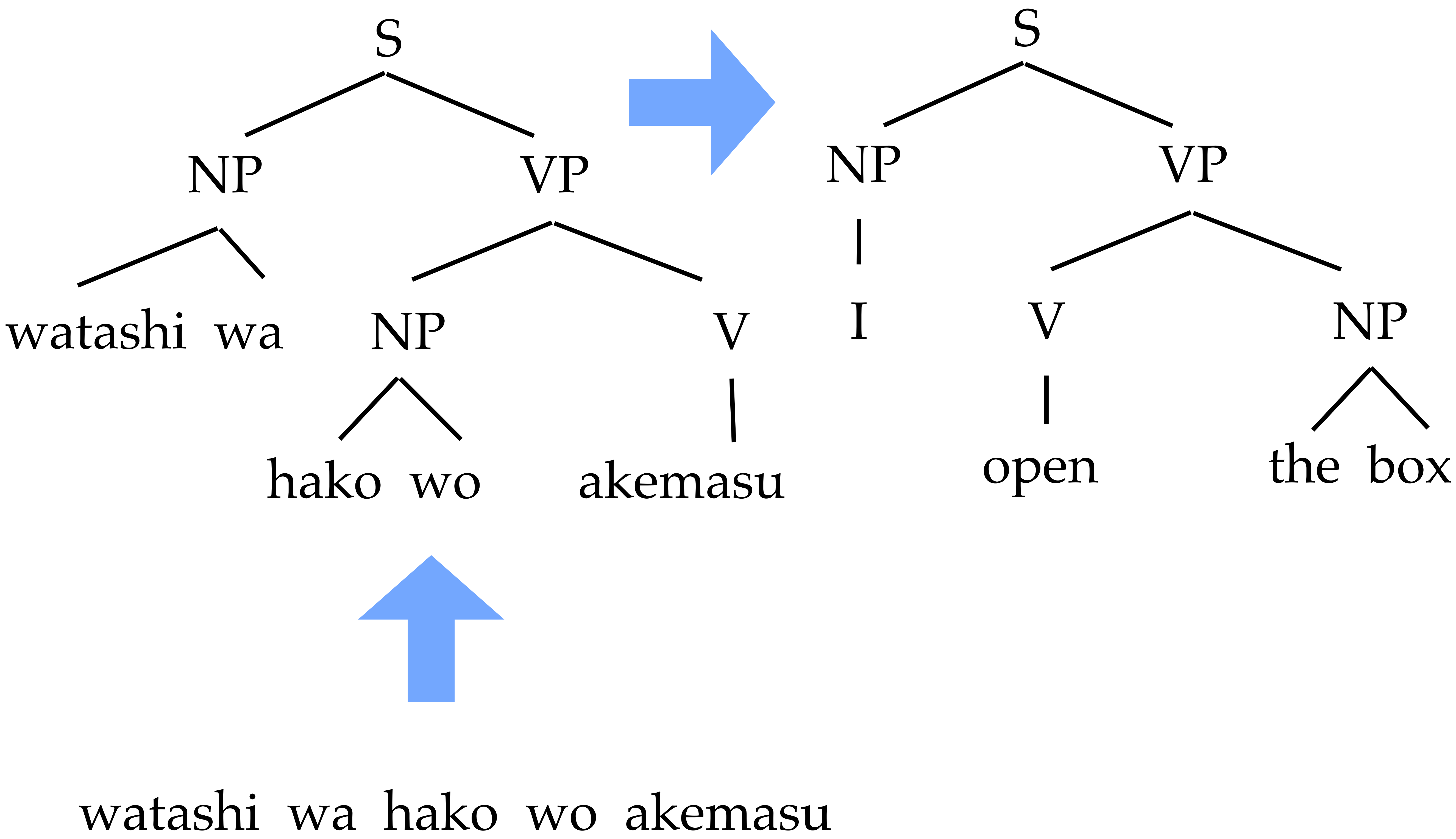

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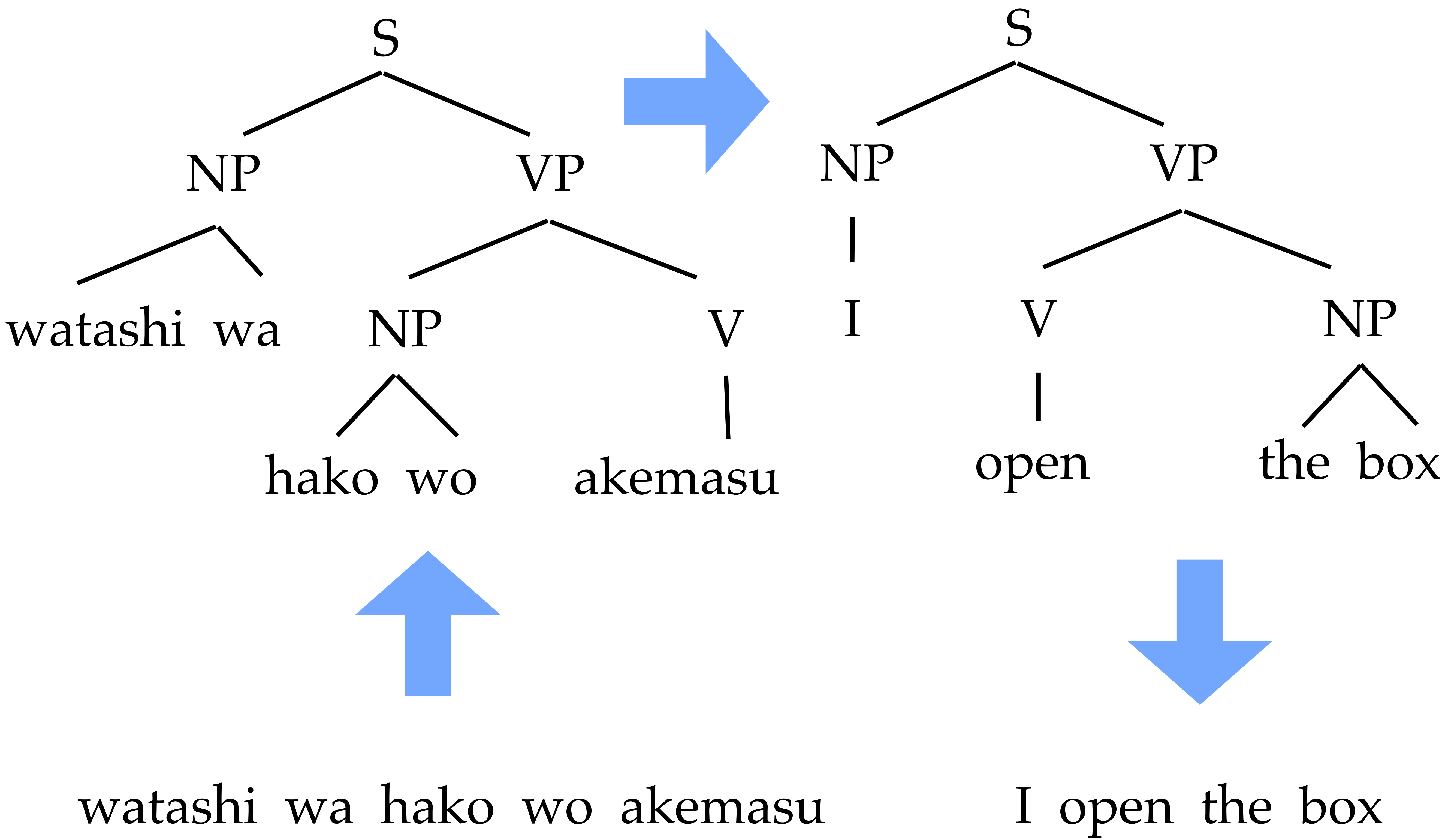
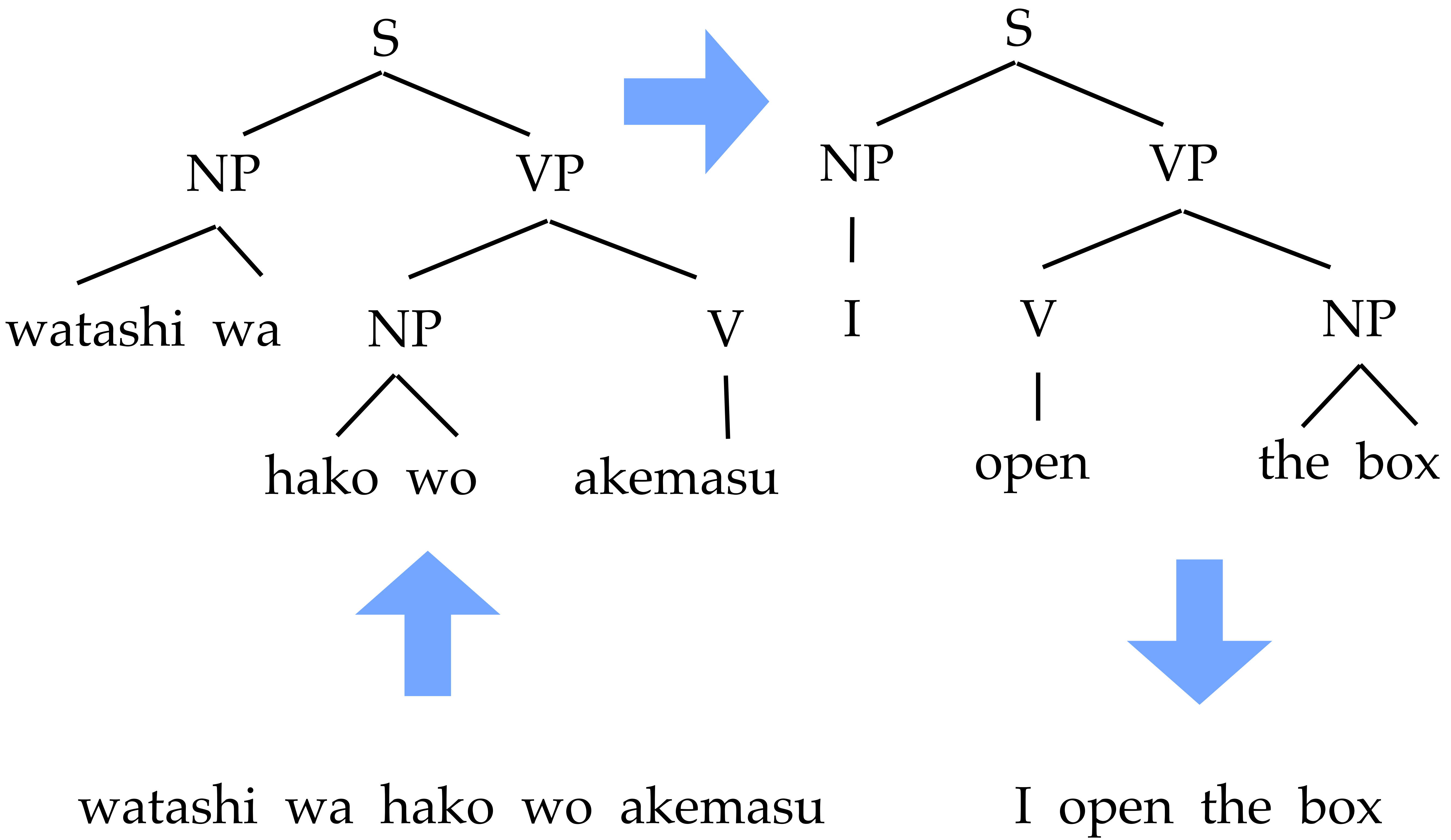


Syntax-based translation

$I \circ G \circ L$ G is a weighted *pushdown assembler* (Aho and Ullman 1969)



Context-Free Parsing

Context-Free Parsing

NN → duck

NP → PRP$ NN

PRP → her

PRP → I

PRP$ → her

S → PRP VP

SBAR → PRP VB

VB → duck

VP → VBD NP

VP → VBD SBAR

VBD → saw

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I_1 saw_2 her_3 $duck_4$

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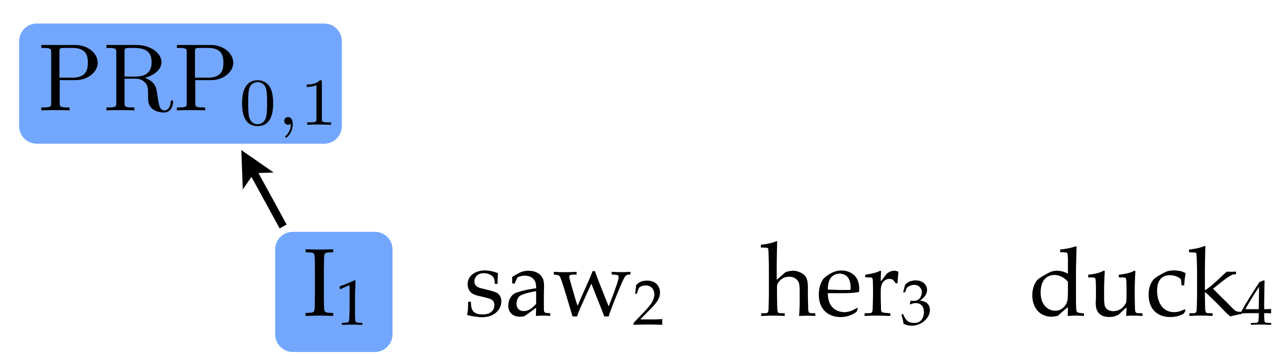

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$\mathrm{PRP}_{0,1}$

↖

I_1 saw_2 her_3 duck_4

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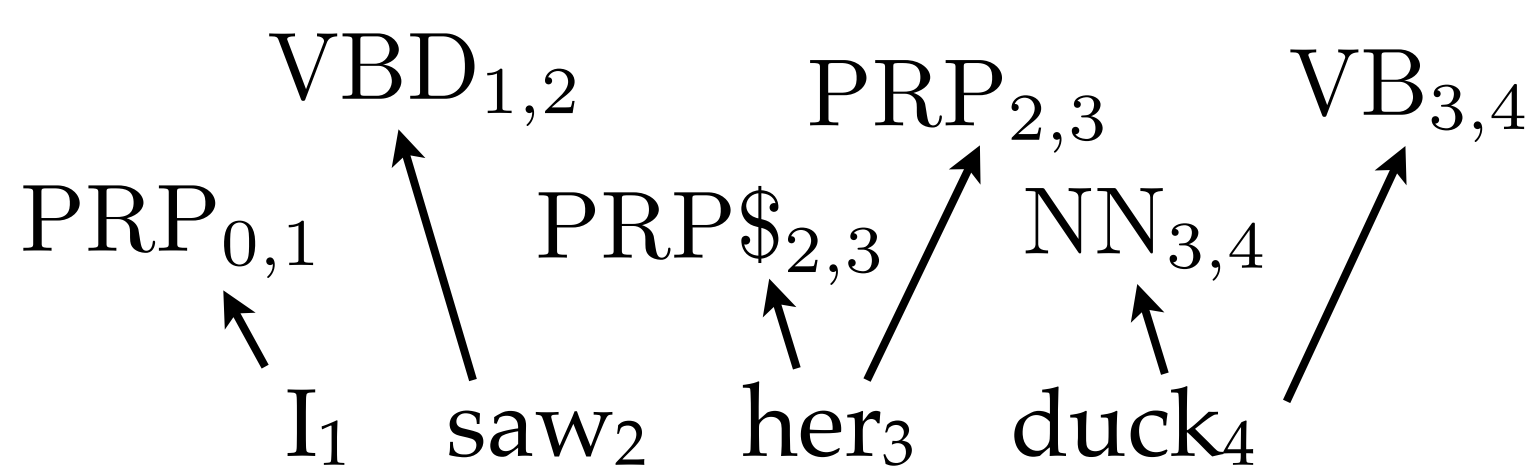

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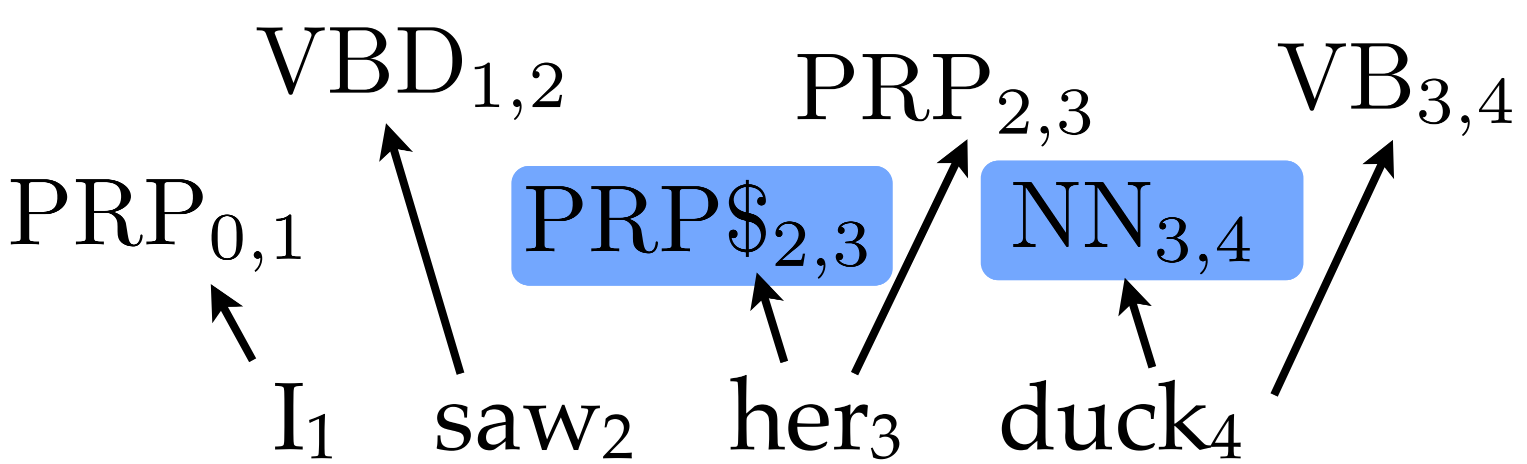

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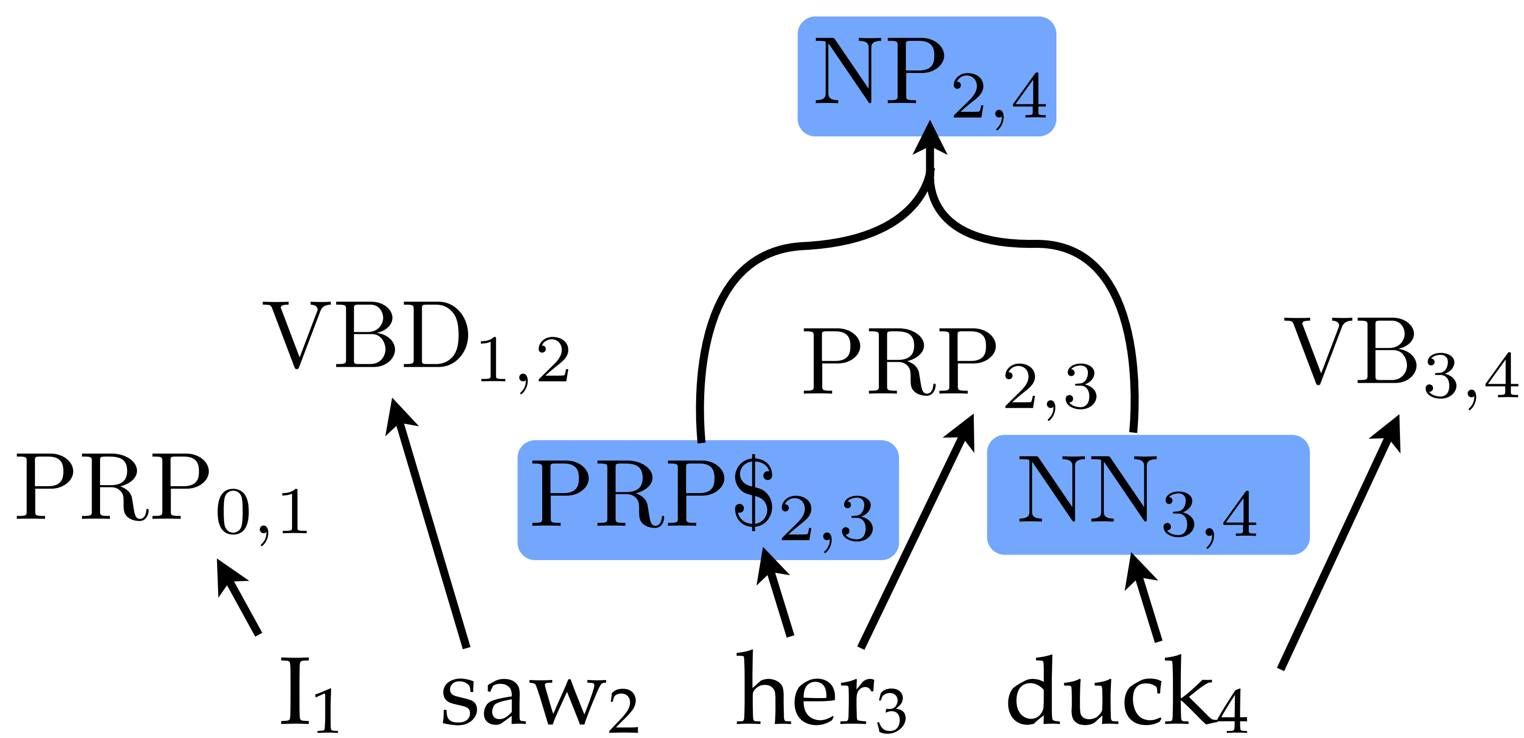

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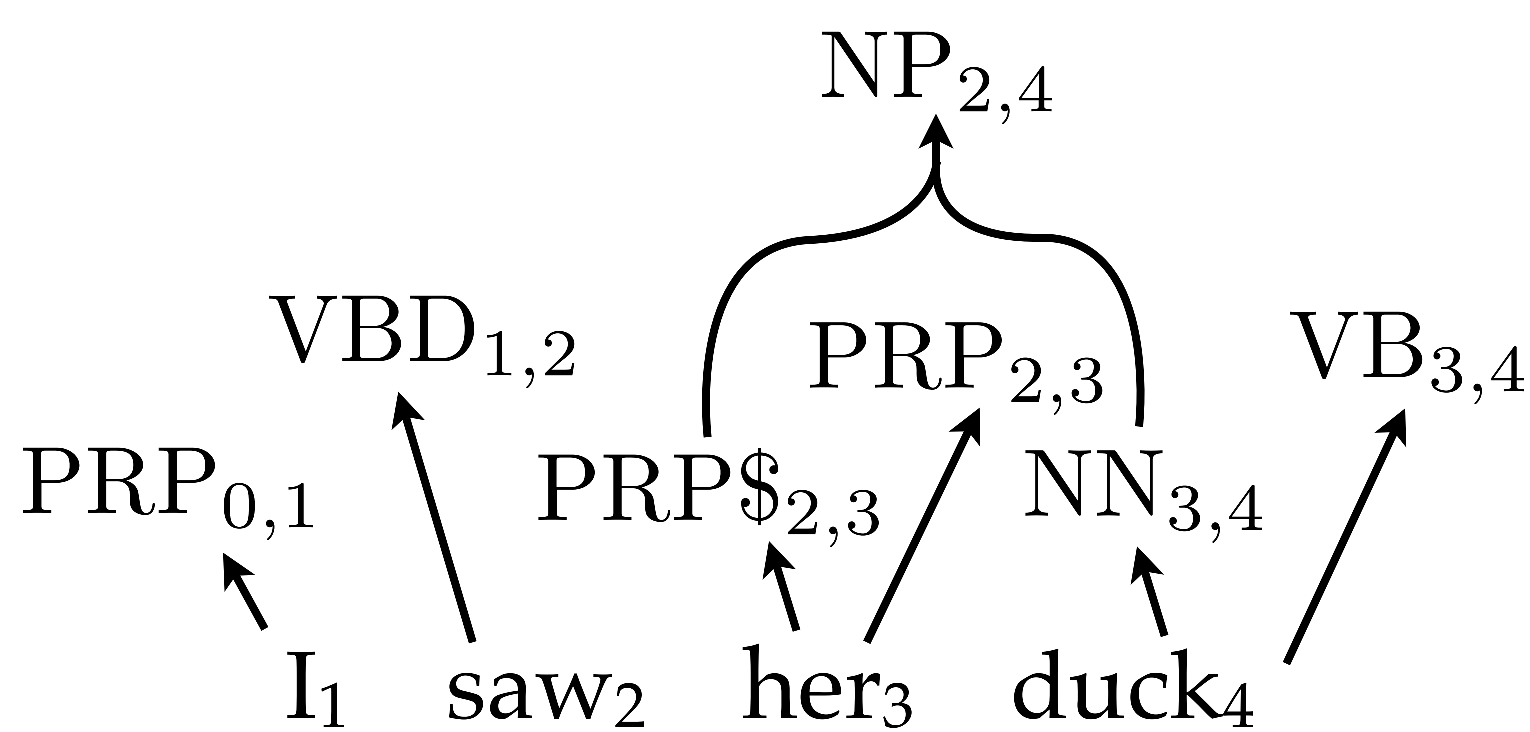

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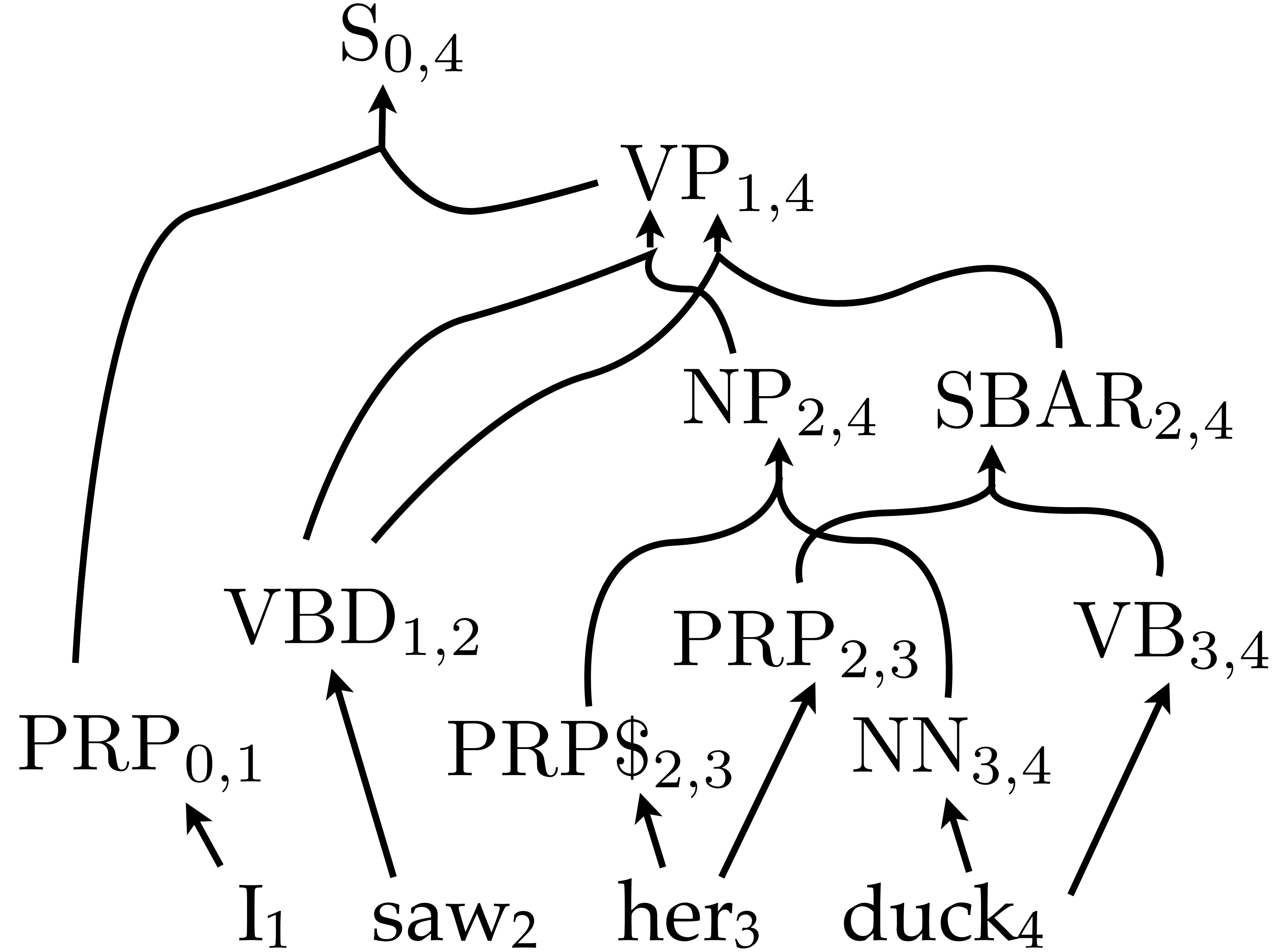

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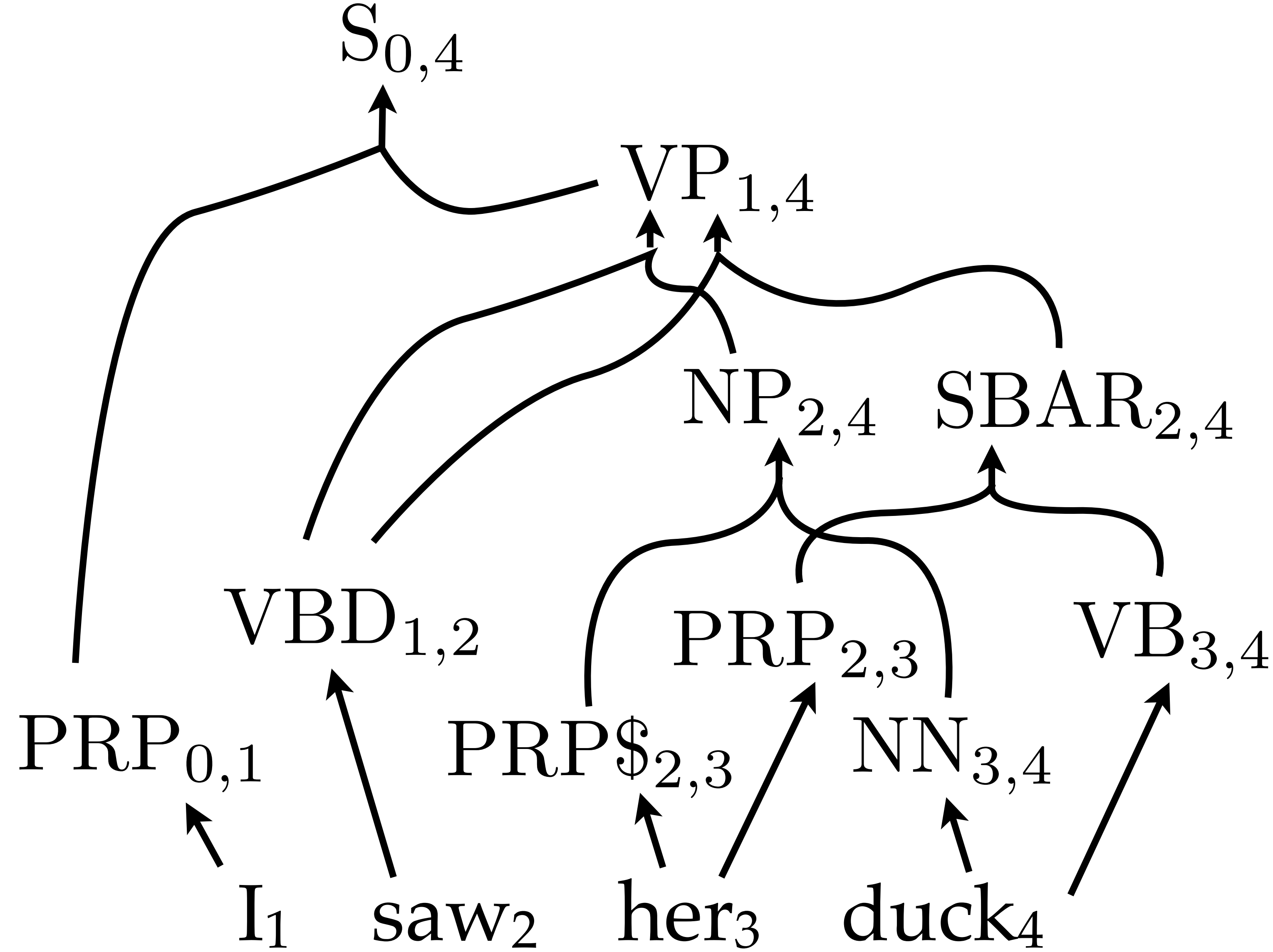

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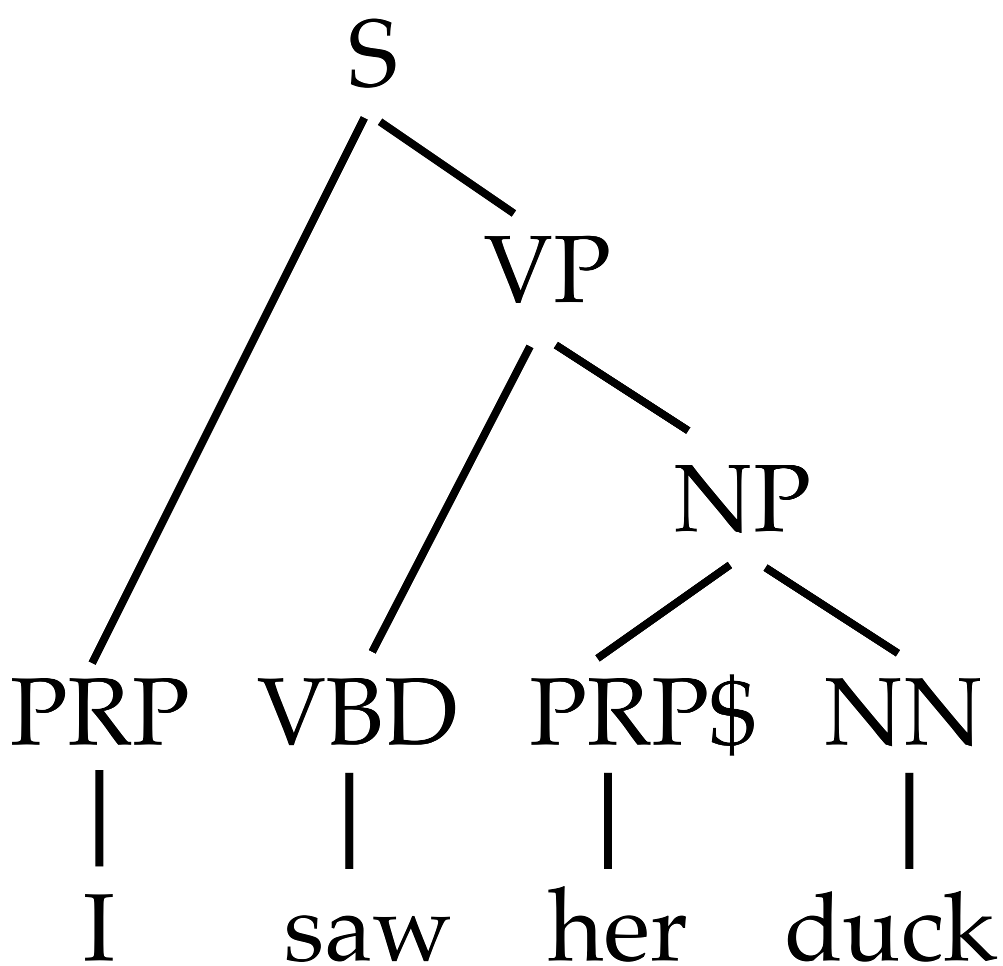

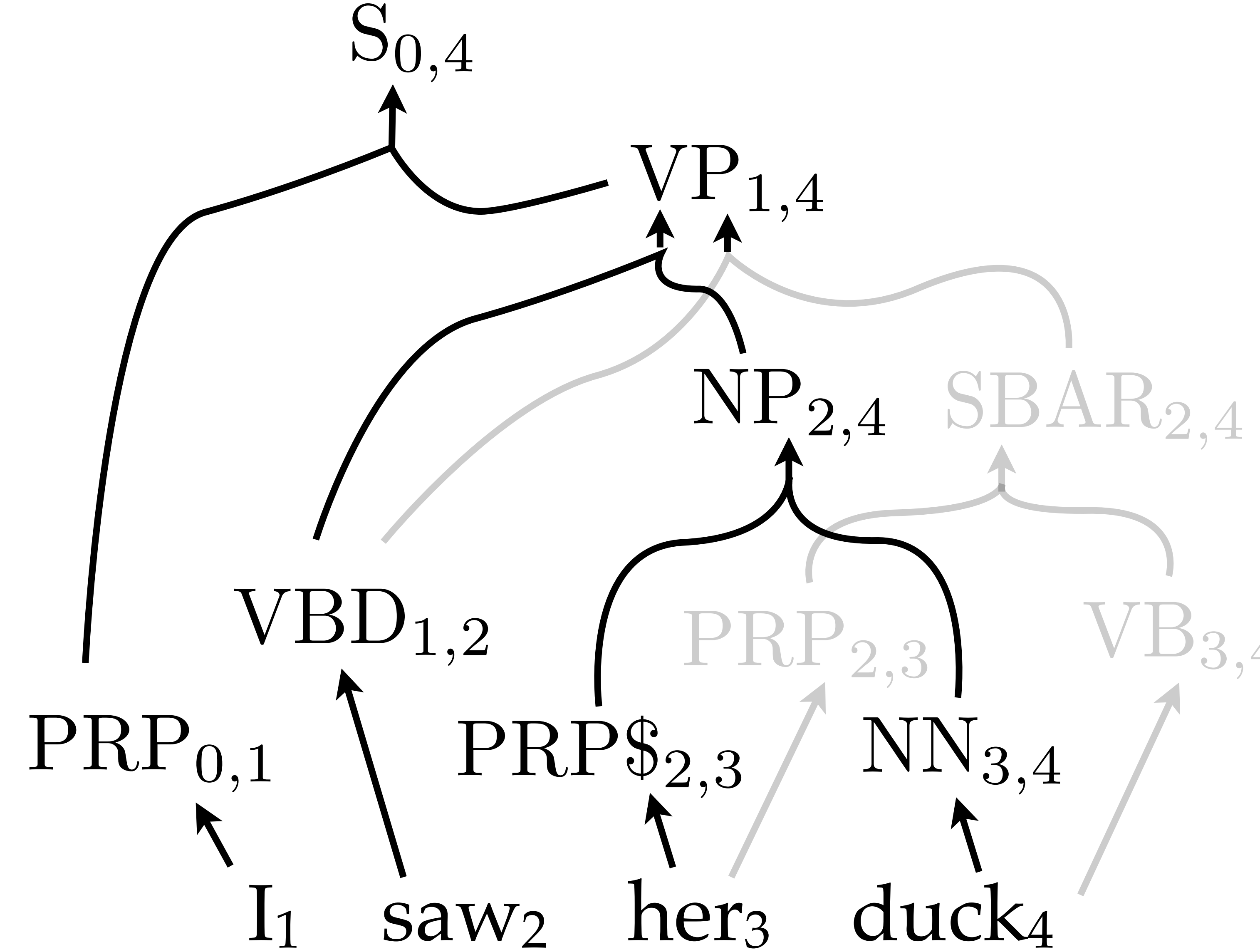

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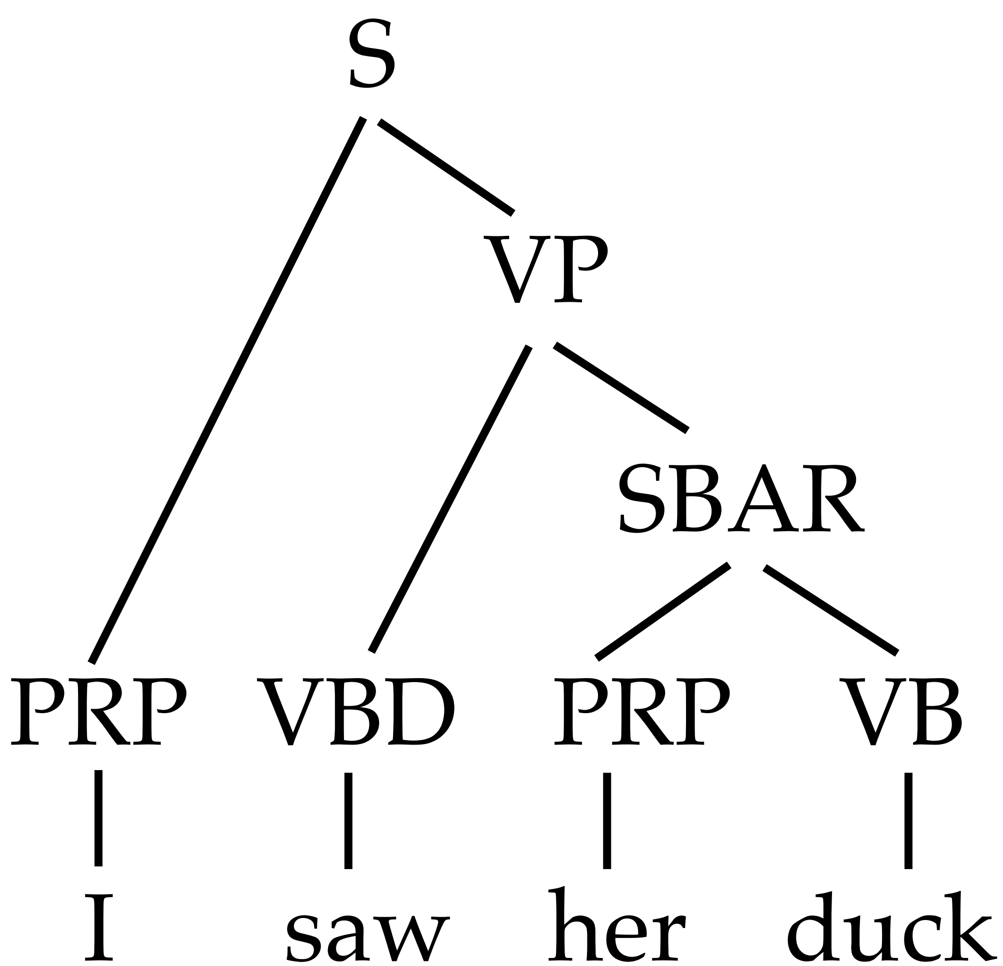
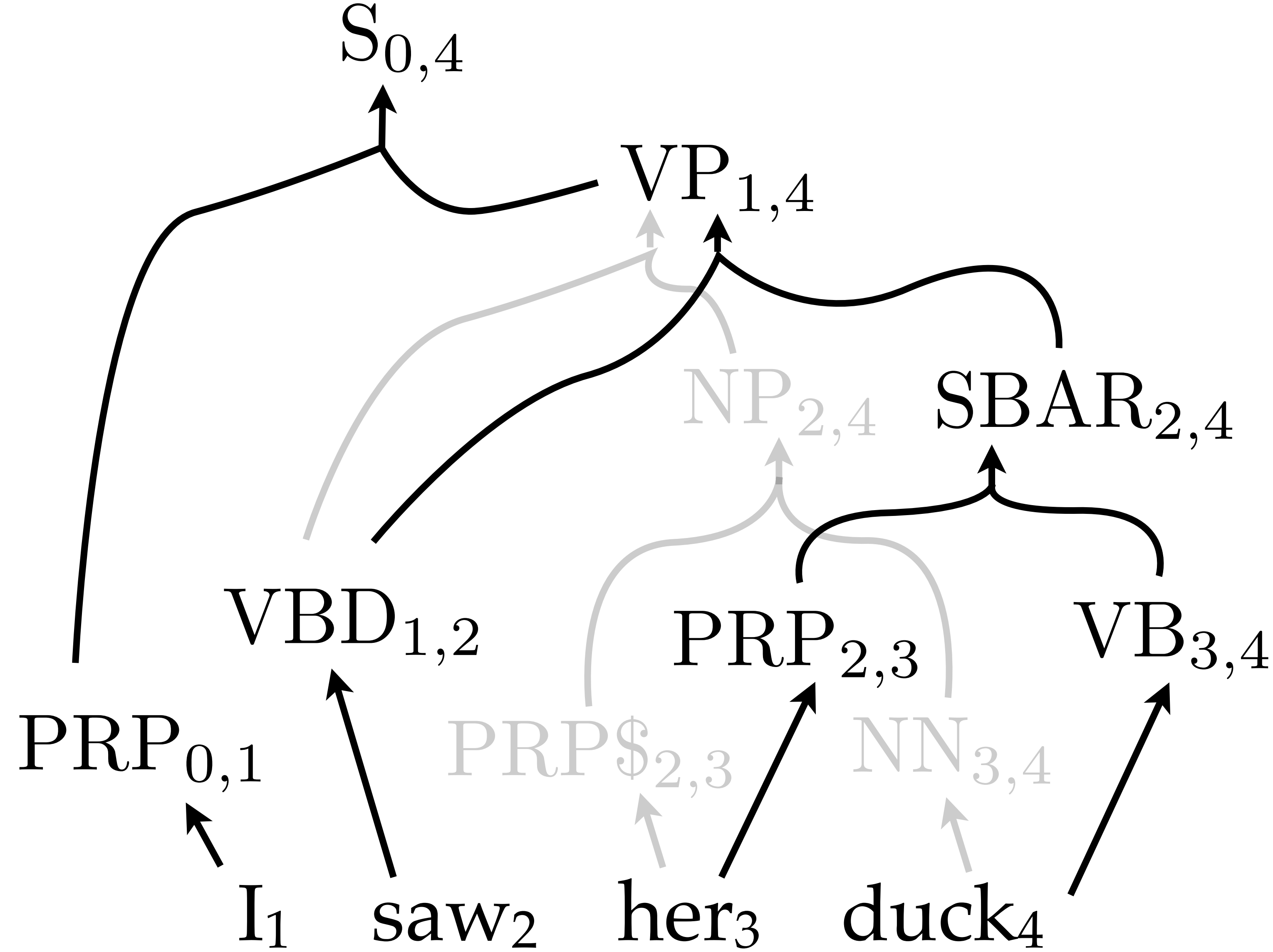
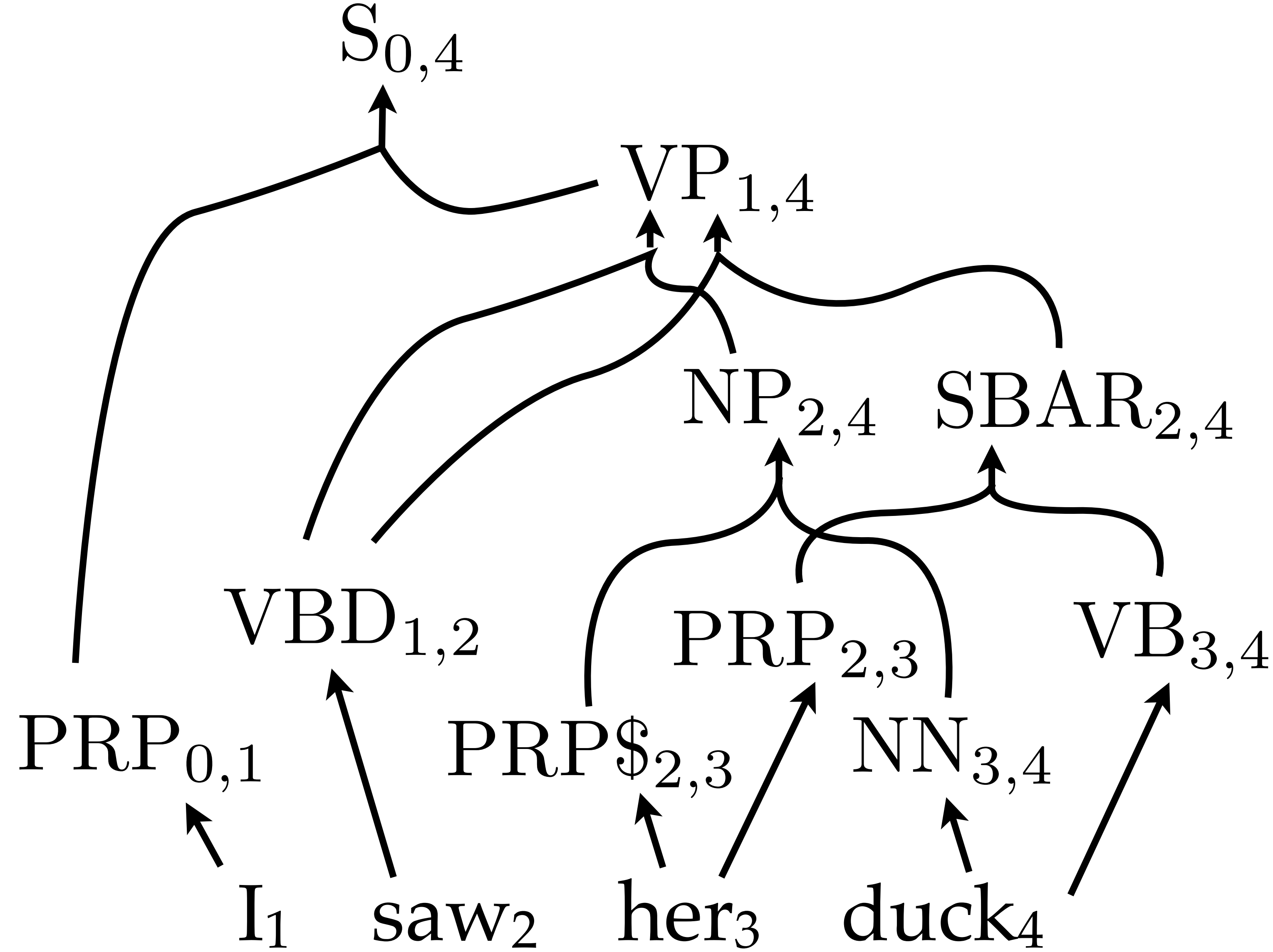




Context-free parsing

... is intersection (Lang 1994)



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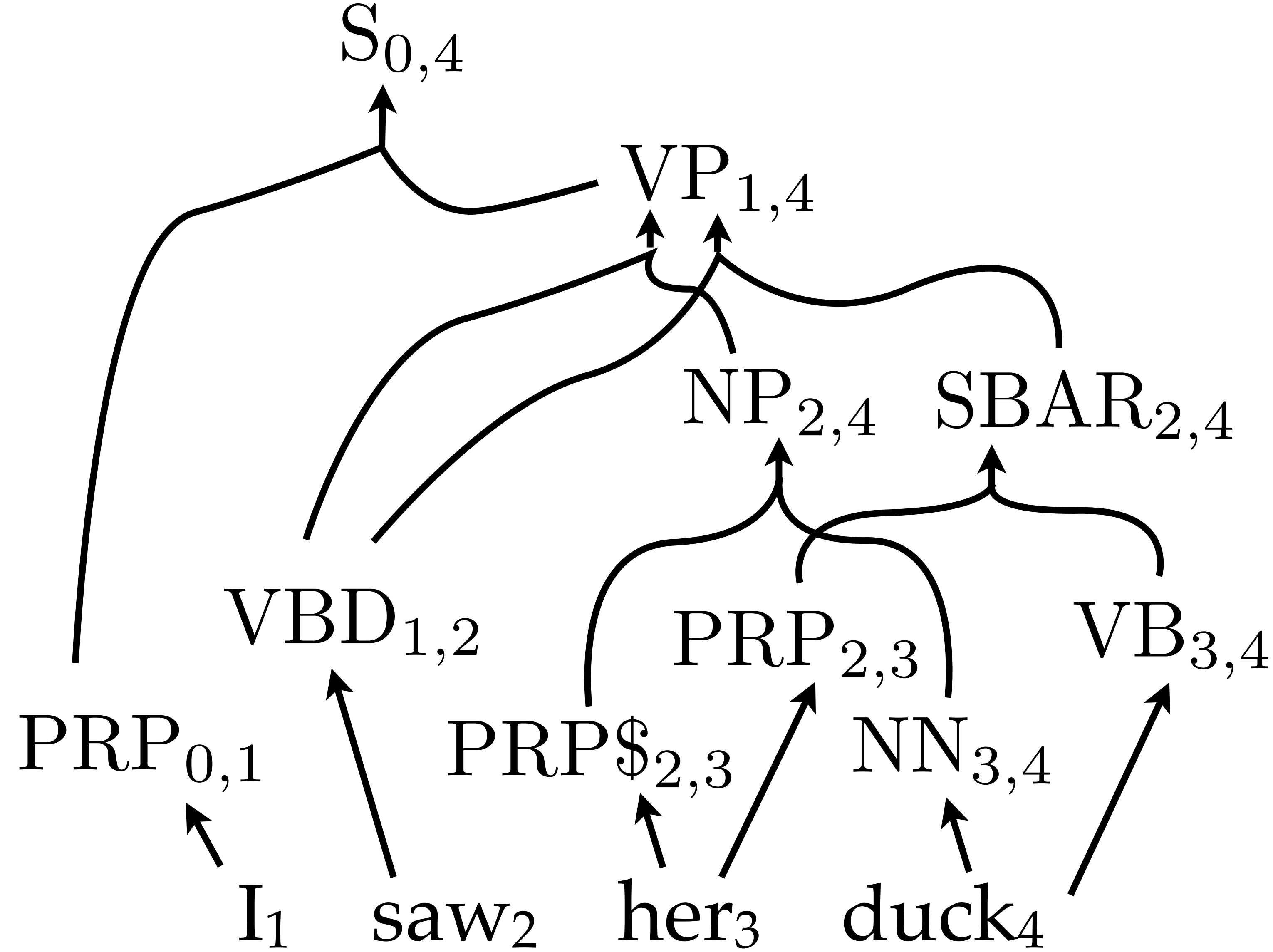
Bar-Hillel et al., 1961

Given CFG $\mathcal{G}$ and NFA $\mathcal{D}$,

Construct grammar $\mathcal{G}'$:



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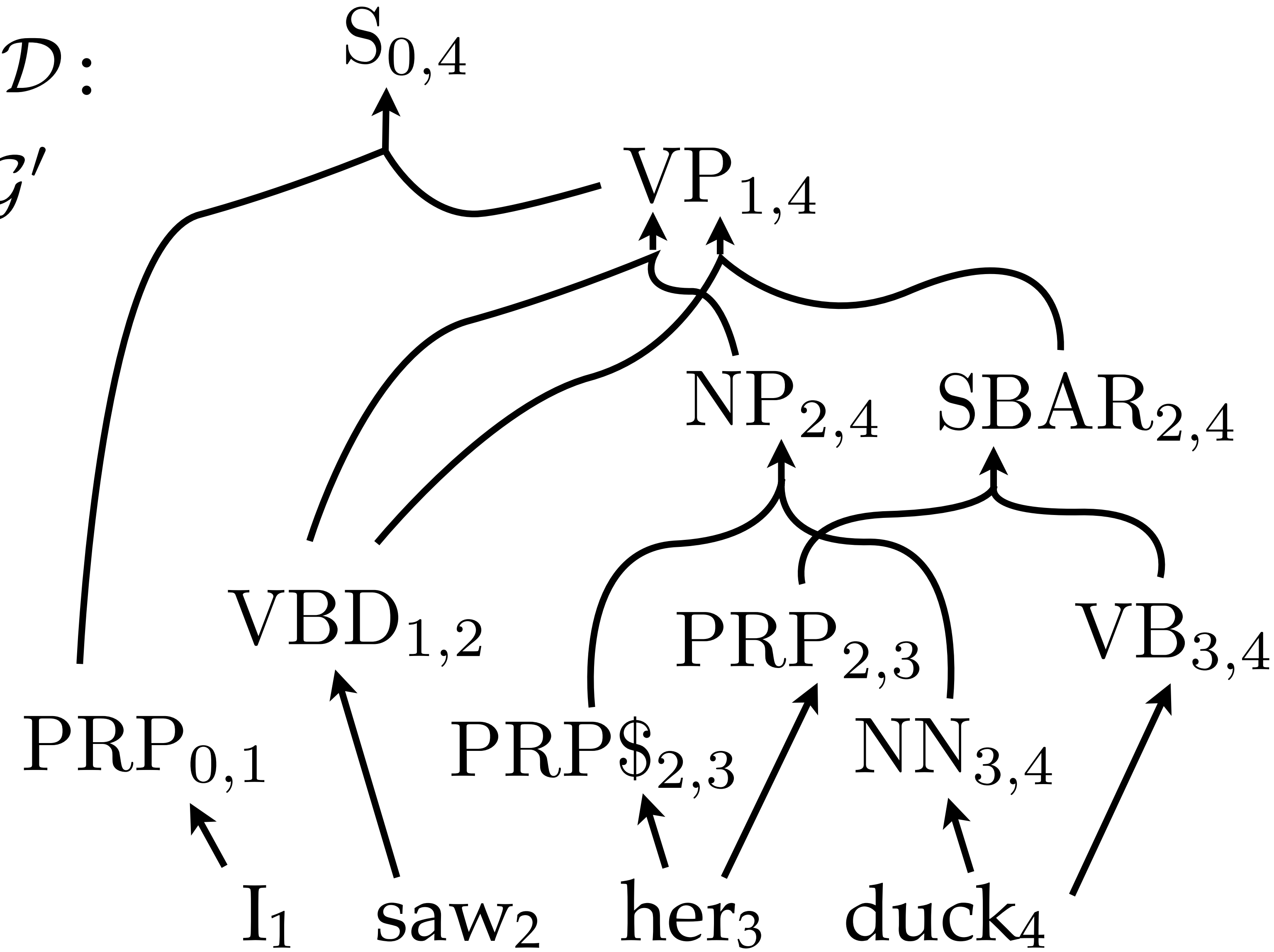
Given CFG $\mathcal{G}$ and NFA $\mathcal{D}$,
Construct grammar $\mathcal{G}'$:

For every rule $A \rightarrow BC$ in $\mathcal{G}$
and three states q, r, s in $\mathcal{D}$:

Add $A_{q,s} \rightarrow B_{q,r}C_{r,s}$ to $\mathcal{G}'$



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For every rule $A \rightarrow w$ in $\mathcal{G}$
and $r, q|r \in \delta(q, w)$ in $\mathcal{D}$:
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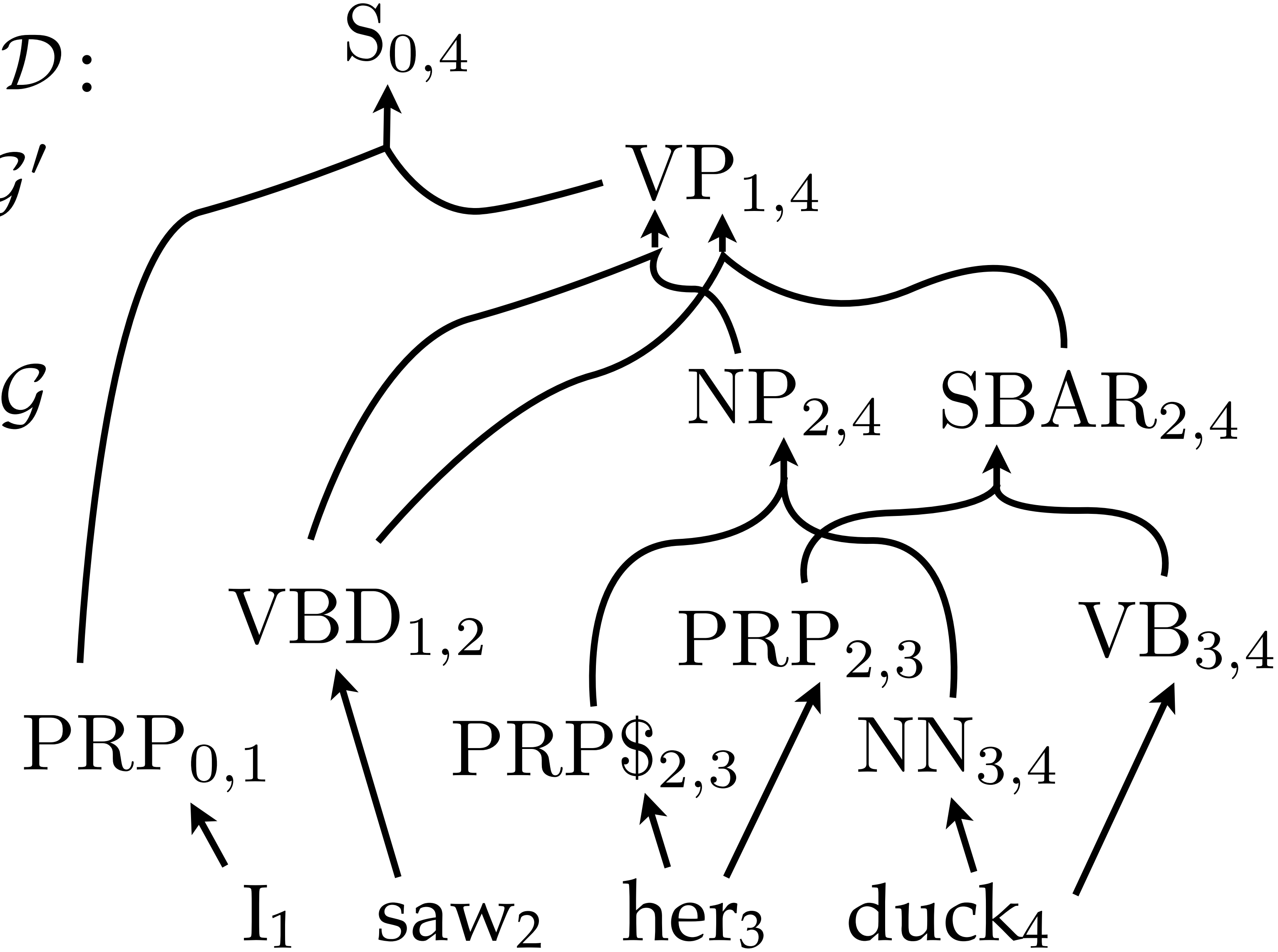

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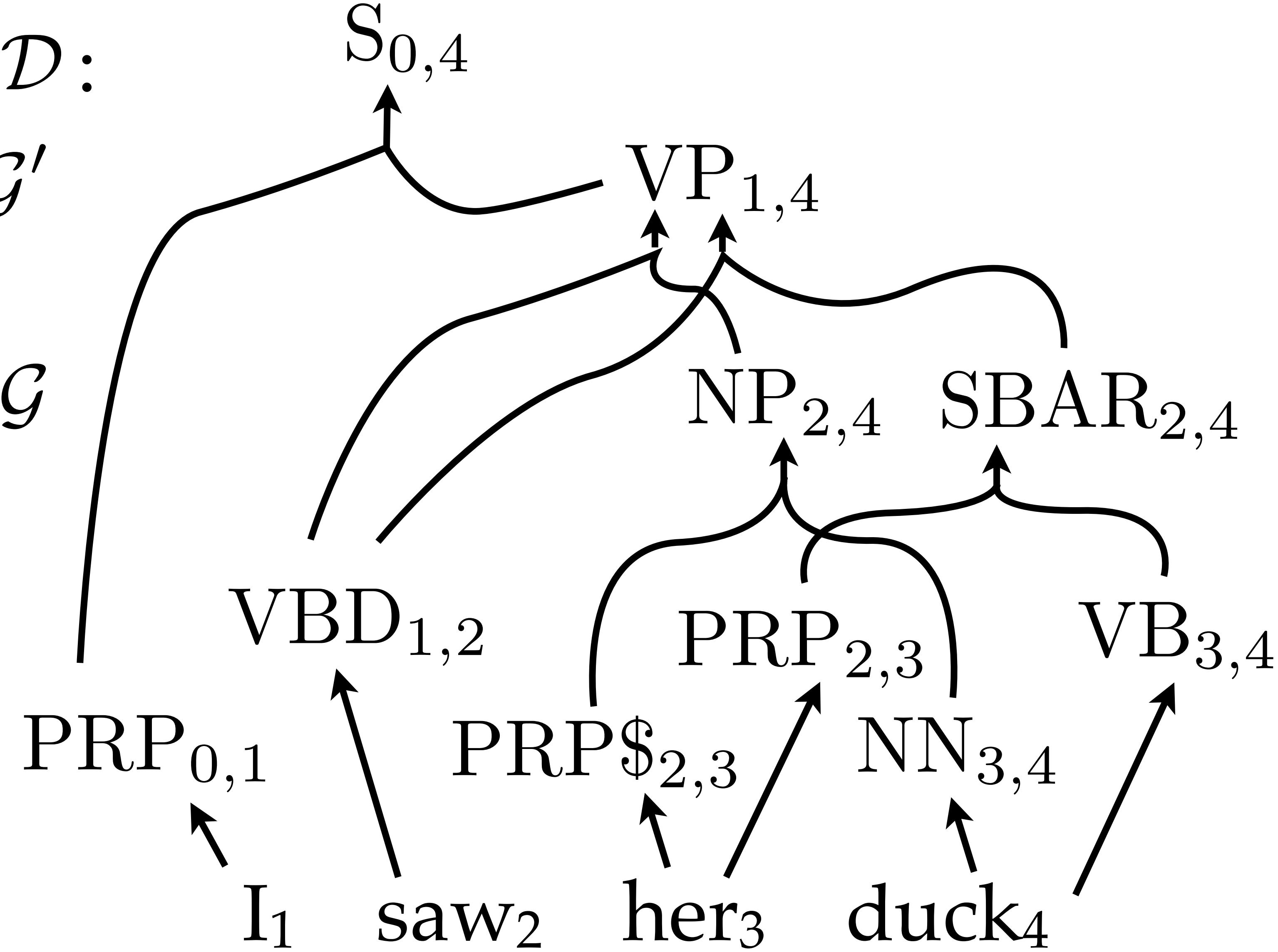
For every state $q \in F$
Add $S \rightarrow S_{q_0,q}$ to $\mathcal{G}'$



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$$L(\mathcal{G}') = L(\mathcal{G}) \cap L(\mathcal{D})$$

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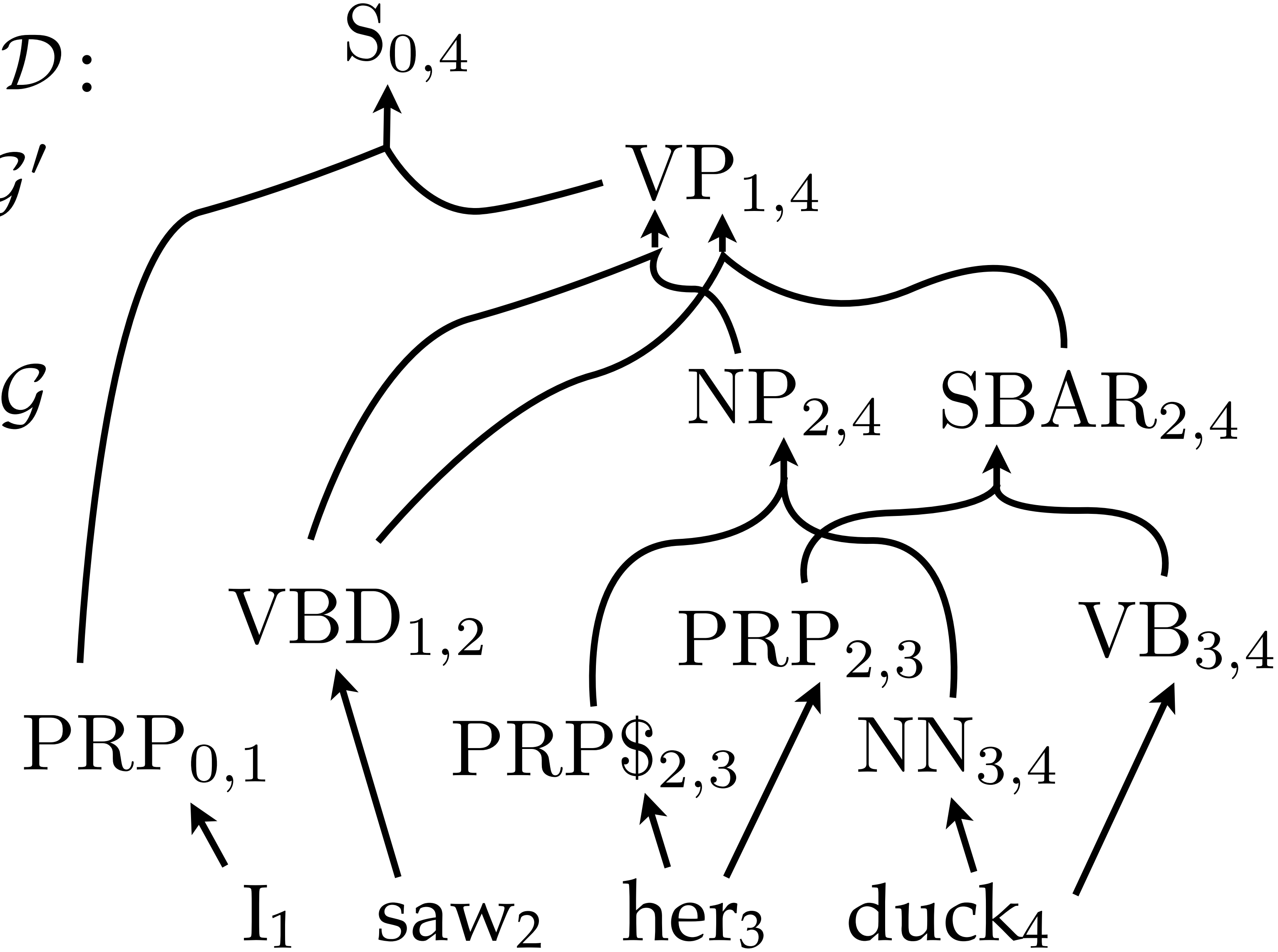

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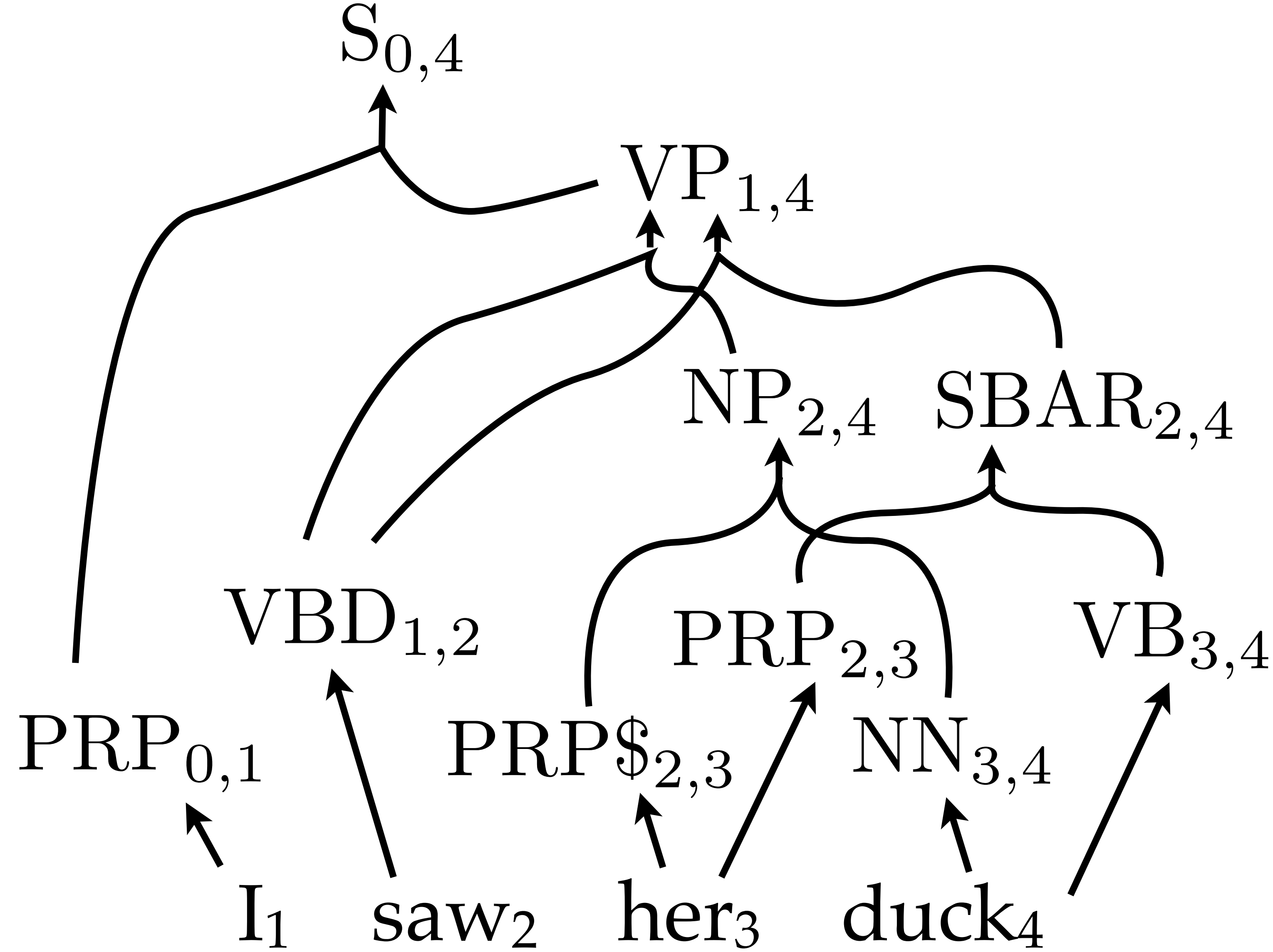

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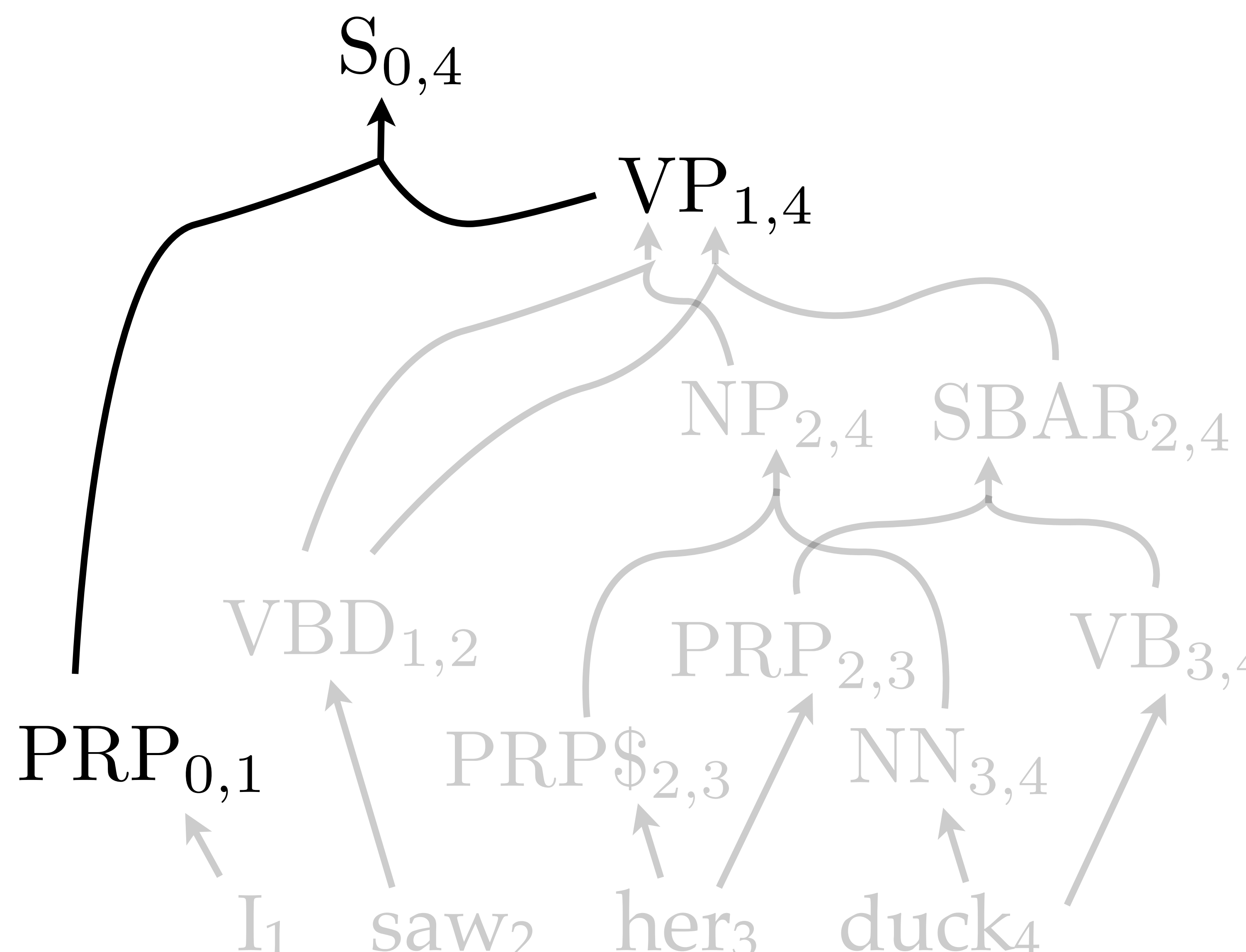
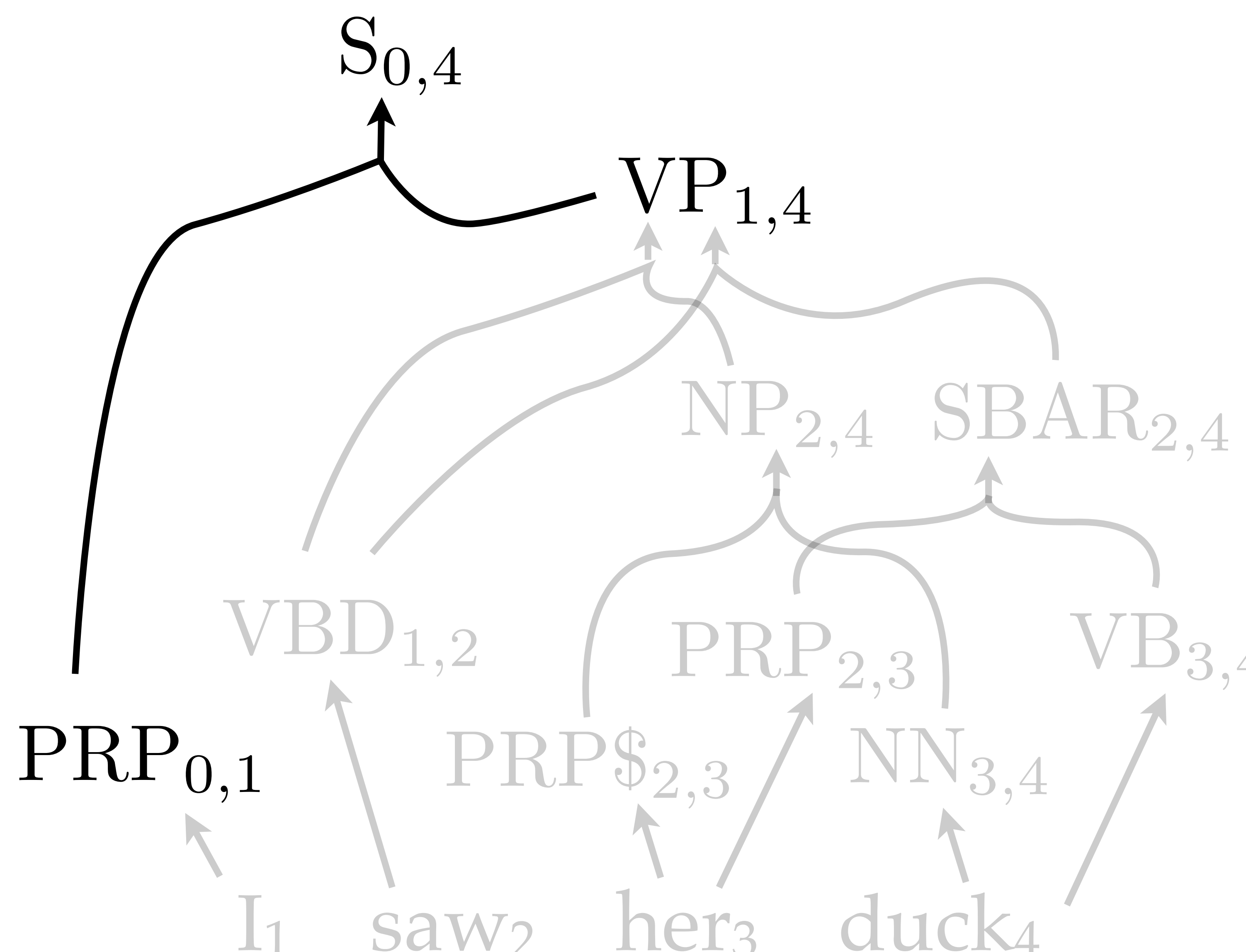


Context-free parsing

... is intersection (Lang 1994)

$S_{0,4} \rightarrow PRP_{0,1}\ VP_{1,4}$



Context-free parsing

... is intersection (Lang 1994)

$NN_{3,4} \rightarrow$ duck

$NP_{2,4} \rightarrow PRP\$_{2,3}\ NN_{3,4}$

$PRP_{2,3} \rightarrow$ her

$PRP_{0,1} \rightarrow$ I

$PRP\$_{2,3} \rightarrow$ her

$S_{0,4} \rightarrow PRP_{0,1}\ VP_{1,4}$

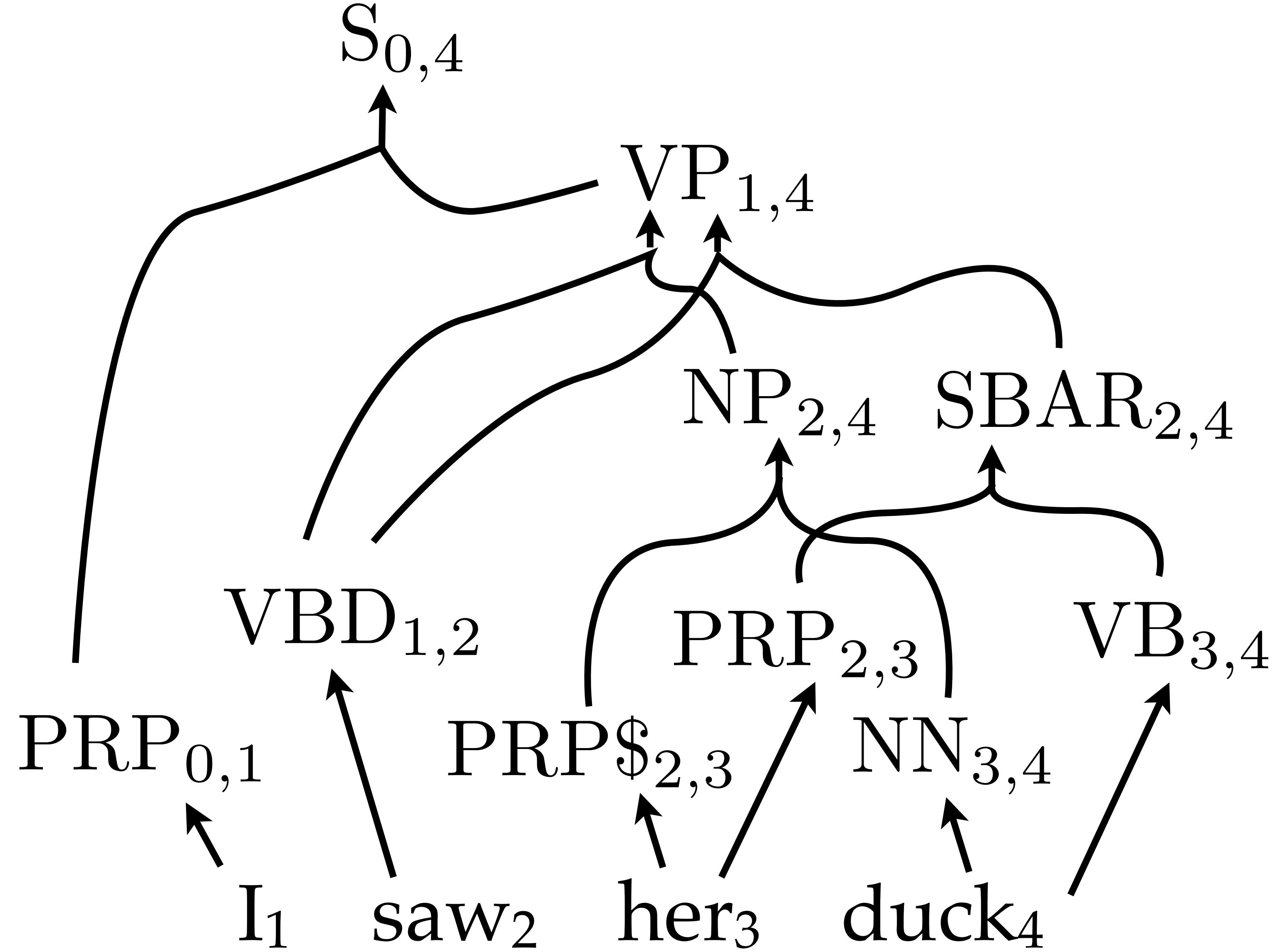
$SBAR_{2,4} \rightarrow PRP_{2,3}\ VB_{3,4}$

$VB_{3,4} \rightarrow$ duck

$VP_{1,4} \rightarrow VBD_{1,2}\ NP_{2,4}$

$VP_{1,4} \rightarrow VBD_{1,2}\ SBAR_{2,4}$

$VBD_{1,2} \rightarrow$ saw



Context-free parsing

... is translation (Satta 2005)

$NN_{3,4} \rightarrow$ duck	$NN_{3,4} \rightarrow$ pato
$NP_{2,4} \rightarrow PRP\$_{2,3}\ NN_{3,4}$	$NP_{2,4} \rightarrow PRP\$_{2,3}\ NN_{3,4}$
$PRP_{2,3} \rightarrow$ her	$PRP_{2,3} \rightarrow$ su
$PRP_{0,1} \rightarrow$ I	$PRP_{0,1} \rightarrow$ yo
$PRP\$_{2,3} \rightarrow$ her	$PRP\$_{2,3} \rightarrow$ ella
$S_{0,4} \rightarrow PRP_{0,1}\ VP_{1,4}$	$S_{0,4} \rightarrow PRP_{0,1}\ VP_{1,4}$
$SBAR_{2,4} \rightarrow PRP_{2,3}\ VB_{3,4}$	$SBAR_{2,4} \rightarrow PRP_{2,3}\ VB_{3,4}$
$VB_{3,4} \rightarrow$ duck	$VB_{3,4} \rightarrow$ agacharse
$VP_{1,4} \rightarrow VBD_{1,2}\ NP_{2,4}$	$VP_{1,4} \rightarrow VBD_{1,2}\ NP_{2,4}$
$VP_{1,4} \rightarrow VBD_{1,2}\ SBAR_{2,4}$	$VP_{1,4} \rightarrow VBD_{1,2}\ SBAR_{2,4}$
$VBD_{1,2} \rightarrow$ saw	$VBD_{1,2} \rightarrow$ vi

Context-free parsing

... is translation (Satta 2005)

$NN_{3,4} \rightarrow$ duck	$NN_{3,4} \rightarrow$ pato
$NP_{2,4} \rightarrow PRP\$_{2,3}\ NN_{3,4}$	$NP_{2,4} \rightarrow PRP\$_{2,3}\ NN_{3,4}$
$PRP_{2,3} \rightarrow$ her	$PRP_{2,3} \rightarrow$ su
$PRP_{0,1} \rightarrow$ I	$PRP_{0,1} \rightarrow$ yo
$PRP\$_{2,3} \rightarrow$ her	$PRP\$_{2,3} \rightarrow$ ella
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$VB_{3,4} \rightarrow$ duck	$VB_{3,4} \rightarrow$ agacharse
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$VP_{1,4} \rightarrow VBD_{1,2}\ SBAR_{2,4}$	$VP_{1,4} \rightarrow VBD_{1,2}\ SBAR_{2,4}$
$VBD_{1,2} \rightarrow$ saw	$VBD_{1,2} \rightarrow$ vi

yo vi ella agacharse

yo vi su pato

Context-free parsing

... is translation

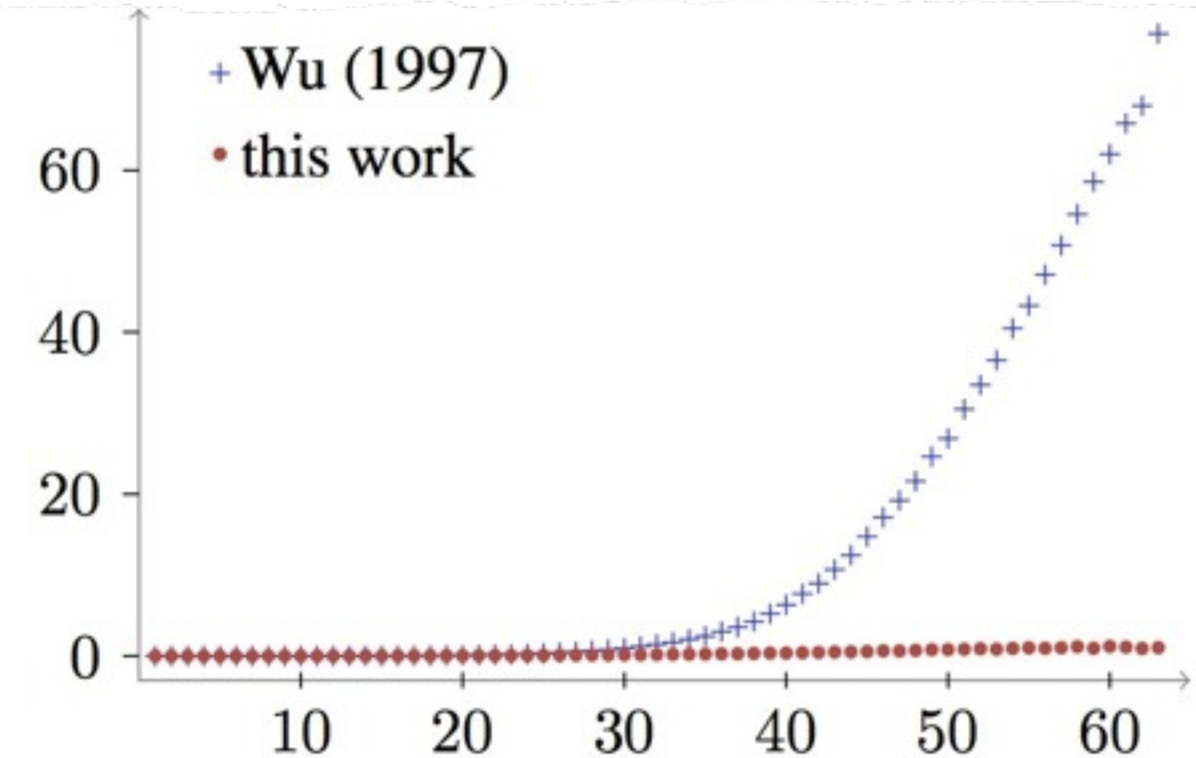


Figure 3: Average synchronous parser run-time (in seconds) as a function of Arabic sentence length (in words).

source: Dyer 2010

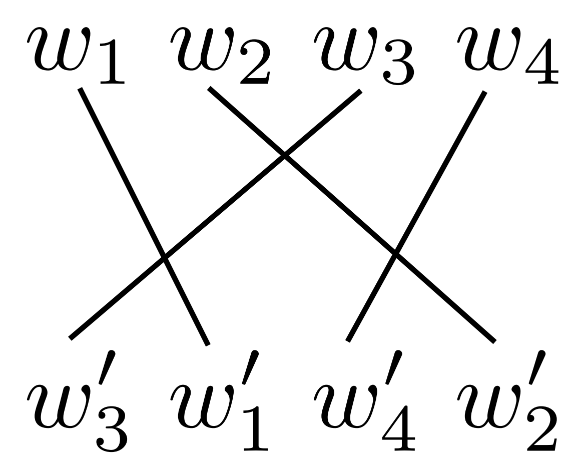
Not all permutations are expressible in SCFG

Aho and Ullman 1969; Wu 1997

A rank-2 SCFG:

$$A \rightarrow A_1 A_2, A_1 A_2$$
$$A \rightarrow A_1 A_2, A_2 A_1$$
$$A \rightarrow w_1, w'_1$$
$$A \rightarrow w_2, w'_2$$
$$A \rightarrow w_3, w'_3$$
$$A \rightarrow w_4, w'_4$$

Cannot represent alignment:



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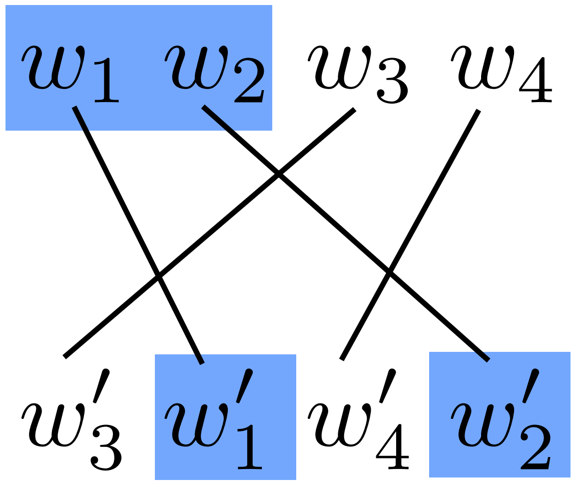

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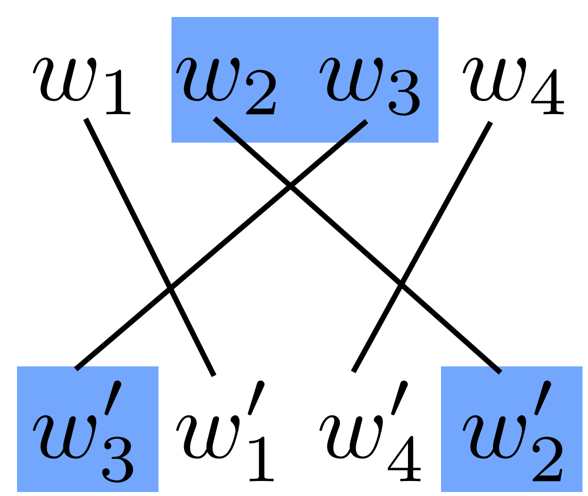

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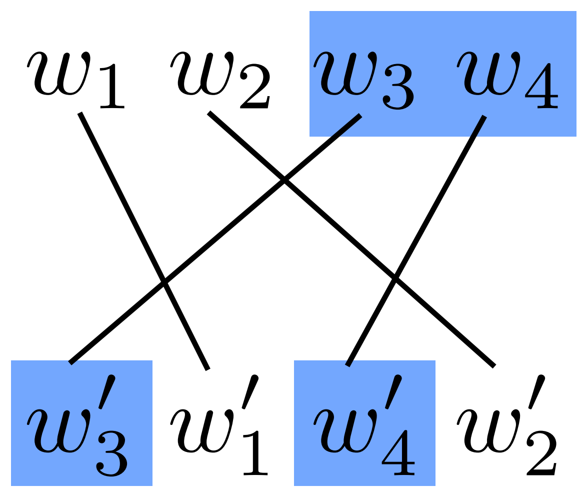

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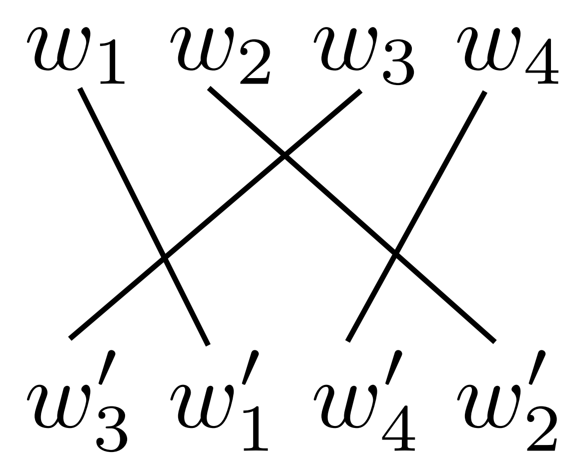

Some problems

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Natural language is not context-free.

Swiss-German, under string homomorphism: $wa^m b^n x c^m d^n y$

Intersect with $wa^* b^* x c^* d^* y$ (Shieber 1985)

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None of these transformations preserve semantics.

Source: *Anna fehlt ihrem Kater*

MT: *Anna is missing her cat* (Jones et al. 2012)

Reference: *Anna's cat is missing her*

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MT: *Anna is missing her cat* (Jones et al. 2012)

Reference: *Anna's cat is missing her*

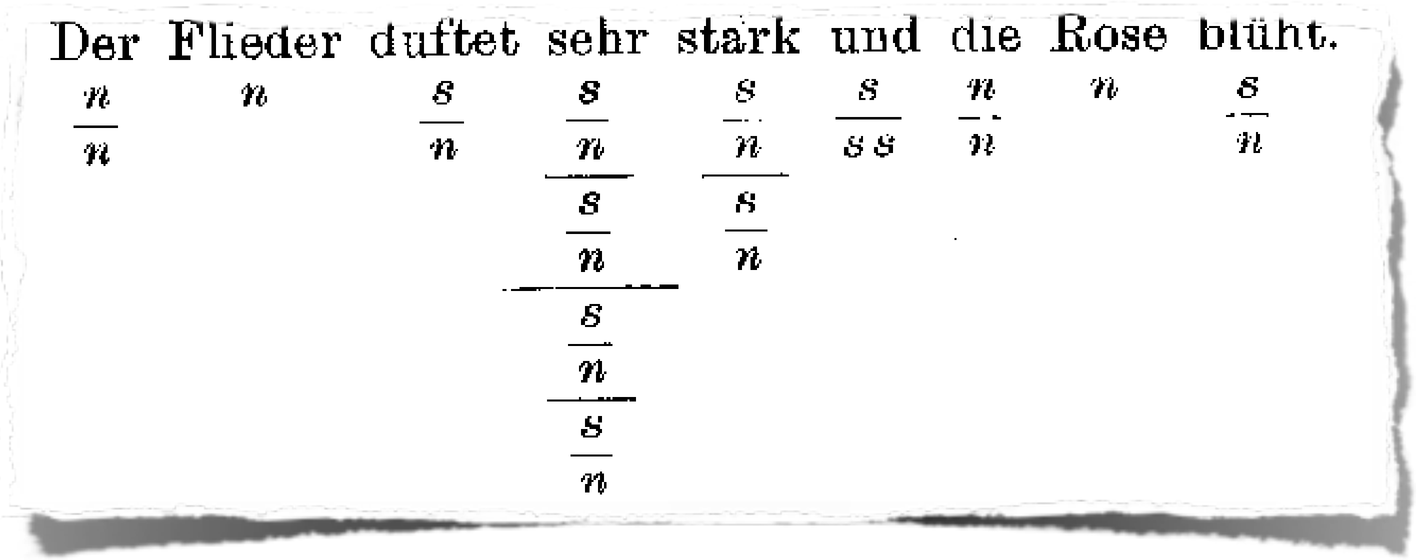
It's (arguably) hard to make SCFG translation models efficient.

Chiang 2005; Huang & Chiang 2007; Venugopal et al. 2007; Petrov et al. 2008; Zhang & Gildea 2008; Hopkins & Langmead 2009; Iglesias et al. 2009, 2011; Huang & Mi 2010; Rush & Collins 2011; Gesmundo et al. 2012

Desiderata for a formal model of translation

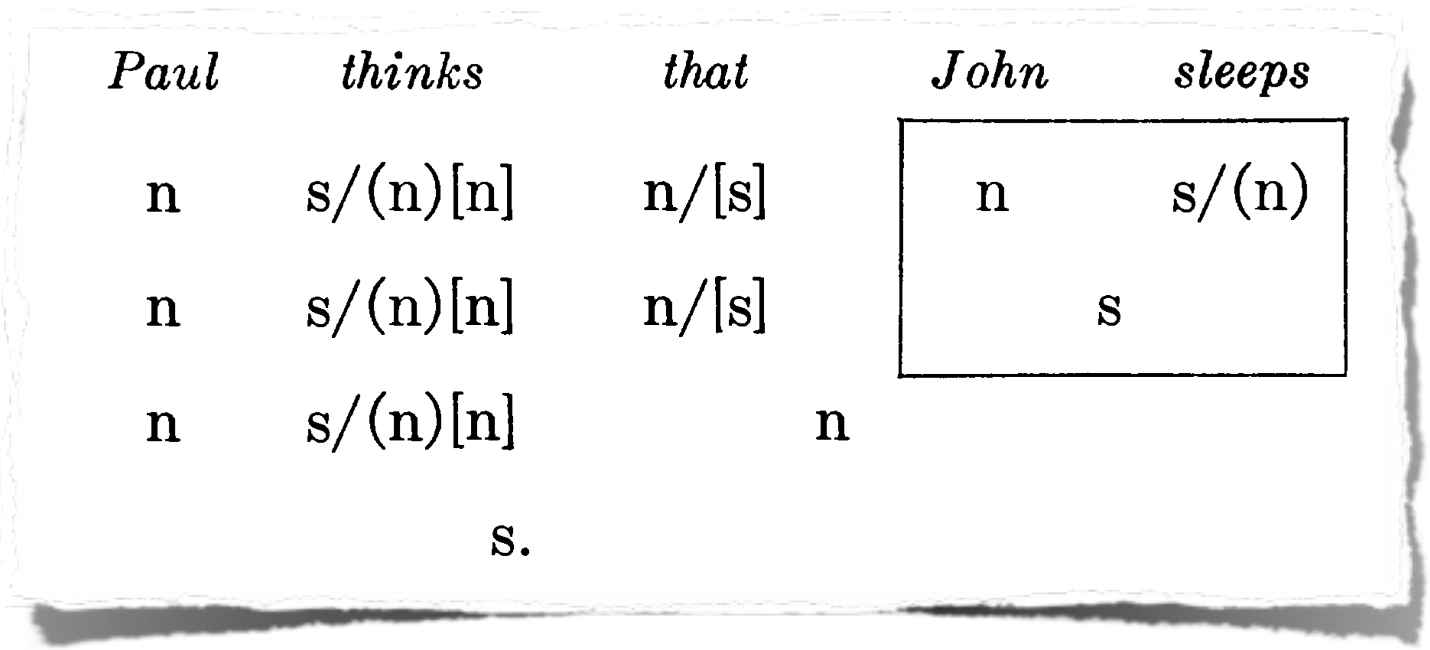
- ☐ Linguistically expressive.
- ☐ Explicit preservation of semantics.
- ☐ Efficient algorithms.
- ☐ Existence of synchronous formalism.

Combinatory categorial grammar



Der	Flieder	duftet	sehr	stark	und	die	Rose	blüht.
$\frac{n}{n}$	n	$\frac{s}{n}$	$\frac{\frac{s}{n}/\frac{s}{n}}{\frac{s}{n}/\frac{s}{n}}$	$\frac{s}{n}/\frac{s}{n}$	$\frac{s}{s\,s}$	$\frac{n}{n}$	n	$\frac{s}{n}$

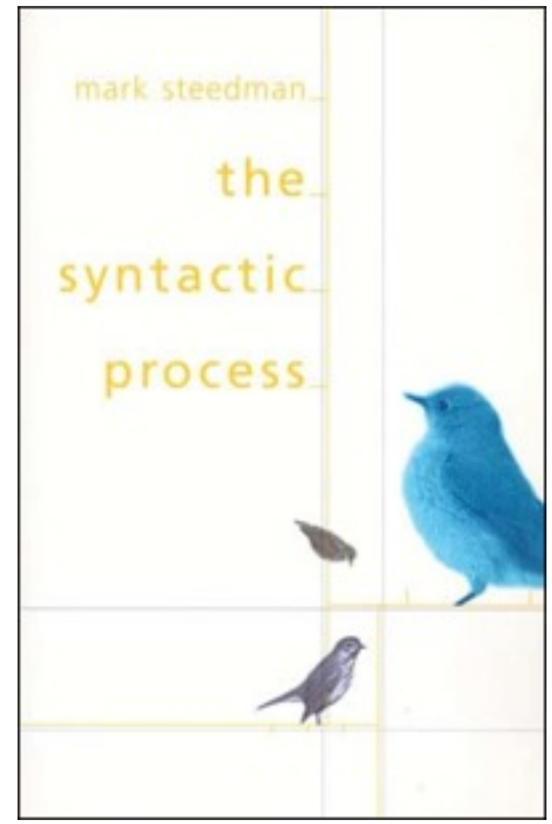
Ajdukiewicz, 1935. *Die syntaktische konnexität*

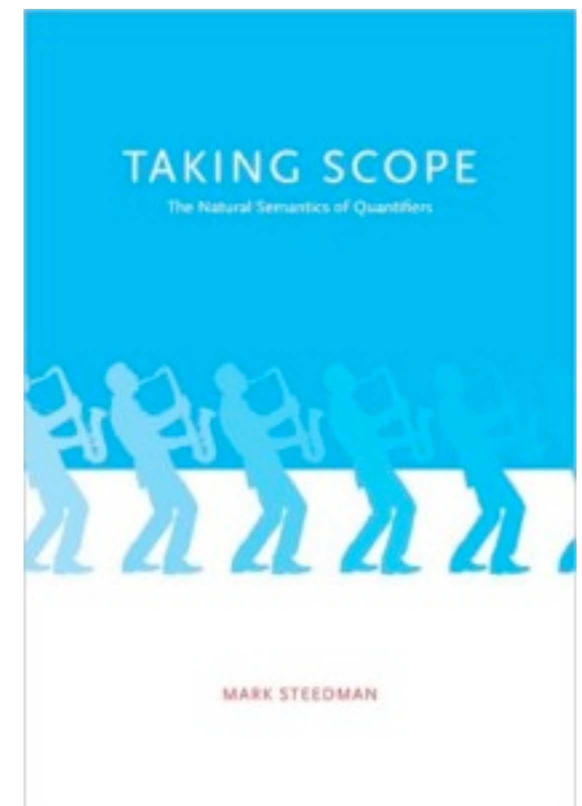


Paul	*thinks*	*that*	*John*	*sleeps*
n	s/(n)[n]	n/[s]	n	s/(n)
n	s/(n)[n]	n/[s]	s	
n	s/(n)[n]	n		
s.				

Bar-Hillel, 1953. *A Quasi-Arithmetical Notation for Syntactic Description*

Steedman, 2000.
The Syntactic Process





Steedman, 2011.
Taking Scope

Categorial grammar

Categorial grammar

A set of **terminals** $\{\text{we}, \text{helped}, \text{Hans}, \text{paint}, \text{the house}\}$

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The complete set of **categories**: if A and B are categories, then A/B and $A\backslash B$ are also categories.

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A **lexicon**: a subset of terminals × categories × lambda terms

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

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functional category

$$\text{we} \vdash \mathsf{NP} : we'$$
$$\text{helped} \vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy$$
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target category

$$\begin{aligned} \text{we} &\vdash \mathsf{NP} : we' \\ \text{helped} &\vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy \\ \text{Hans} &\vdash \mathsf{NP} : Hans' \\ \text{paint} &\vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'\,x \\ \text{the house} &\vdash \mathsf{NP} : house' \end{aligned}$$

Categorial grammar

A set of **terminals** {we, helped, Hans, paint, the house}

A set of **atomic categories** (nonterminals) $\{\mathsf{NP}, \underline{\mathsf{S}}, \mathsf{VP}\}$

The complete set of **categories**: if A and B are categories, then A/B and $A\backslash B$ are also categories.

A **lexicon**: a subset of terminals × categories × lambda terms

argument categories

$$
\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
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\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'\,x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}
$$

Categorial grammar

we helped Hans paint the house

$$
\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\, fxy \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint' x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}
$$

Categorial grammar

$$\dfrac{\text{we}}{\mathsf{NP} : we'} \quad \dfrac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy} \quad \dfrac{\text{Hans}}{\mathsf{NP} : Hans'} \quad \dfrac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'x} \quad \dfrac{\text{the house}}{\mathsf{NP} : house'}$$

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Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

$$\frac{\text{we}}{\mathsf{NP} : we'} \quad \frac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped' fxy} \quad \frac{\text{Hans}}{\mathsf{NP} : Hans'} \quad \frac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint' x} \quad \frac{\text{the house}}{\mathsf{NP} : house'}$$

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primary premise

$$\frac{\text{we}}{\mathsf{NP} : we'} \quad \frac{\text{helped}}{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy} \quad \frac{\text{Hans}}{\mathsf{NP} : Hans'} \quad \frac{\text{paint}}{\mathsf{VP/NP} : \lambda x.paint'\,x} \quad \frac{\text{the house}}{\mathsf{NP} : house'}$$

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secondary premise

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we	helped	Hans	paint	the house
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			$\mathsf{VP} : paint'\,house'$ (>)	

$$
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$\mathsf{NP} : we'$	$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy$	$\mathsf{NP} : Hans'$	$\mathsf{VP/NP} : \lambda x.paint'x$	$\mathsf{NP} : house'$
	$\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,Hans'y$ (>)		$\mathsf{VP} : paint'\,house'$ (>)	

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
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\text{we} &\vdash \mathsf{NP} : we' \\
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Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy$	$\mathsf{NP} : Hans'$	$\mathsf{VP/NP} : \lambda x.paint'x$	$\mathsf{NP} : house'$

helped + Hans (>): $\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f Hans' y$

paint + the house (>): $\mathsf{VP} : paint'\,house'$

(>): $\mathsf{S\backslash NP} : \lambda y.helped'(paint'\,house')Hans'y$

$$
\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}
$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy$	$\mathsf{NP} : Hans'$	$\mathsf{VP/NP} : \lambda x.paint'x$	$\mathsf{NP} : house'$

$$\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'\,f\,Hans'y \quad (>)$$

$$\mathsf{VP} : paint'house' \quad (>)$$

$$\mathsf{S\backslash NP} : \lambda y.helped'(paint'house')Hans'y \quad (>)$$

$$\text{we} \vdash \mathsf{NP} : we'$$

$$\text{helped} \vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy$$

$$\text{Hans} \vdash \mathsf{NP} : Hans'$$

$$\text{paint} \vdash \mathsf{VP/NP} : \lambda x.paint'x$$

$$\text{the house} \vdash \mathsf{NP} : house'$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped' fxy$	$\mathsf{NP} : Hans'$	$\mathsf{VP/NP} : \lambda x.paint' x$	$\mathsf{NP} : house'$

$$\frac{\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped' fxy \quad \mathsf{NP} : Hans'}{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped' f Hans' y} >$$

$$\frac{\mathsf{VP/NP} : \lambda x.paint' x \quad \mathsf{NP} : house'}{\mathsf{VP} : paint' house'} >$$

$$\frac{\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped' f Hans' y \quad \mathsf{VP} : paint' house'}{\mathsf{S\backslash NP} : \lambda y.helped'(paint' house') Hans' y} >$$

$$\frac{\mathsf{NP} : we' \quad \mathsf{S\backslash NP} : \lambda y.helped'(paint' house') Hans' y}{\mathsf{S} : helped'(paint' house') Hans' we'} <$$

$$\begin{aligned} \text{we} &\vdash \mathsf{NP} : we' \\ \text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped' fxy \\ \text{Hans} &\vdash \mathsf{NP} : Hans' \\ \text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint' x \\ \text{the house} &\vdash \mathsf{NP} : house' \end{aligned}$$

Categorial grammar

forward application

$$A/B : f \quad B : g \Rightarrow A : fg$$

backward application

$$B : g \quad A\backslash B : f \Rightarrow A : fg$$

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy$	$\mathsf{NP} : Hans'$	$\mathsf{VP/NP} : \lambda x.paint'x$	$\mathsf{NP} : house'$

$$\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'fHans'y \quad (>)$$

$$\mathsf{VP} : paint'house' \quad (>)$$

$$\mathsf{S\backslash NP} : \lambda y.helped'(paint'house')Hans'y \quad (>)$$

$$\mathsf{S} : helped'(paint'house')Hans'we' \quad (<)$$

$$\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}$$

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy$	$\mathsf{NP} : Hans'$	$\mathsf{VP/NP} : \lambda x.paint'x$	$\mathsf{NP} : house'$

$$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy \quad \mathsf{NP} : Hans' \;\Rightarrow_{>}\; \mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'fHans'y$$

$$\mathsf{VP/NP} : \lambda x.paint'x \quad \mathsf{NP} : house' \;\Rightarrow_{>}\; \mathsf{VP} : paint'house'$$

$$\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'fHans'y \quad \mathsf{VP} : paint'house' \;\Rightarrow_{>}\; \mathsf{S\backslash NP} : \lambda y.helped'(paint'house')Hans'y$$

$$\mathsf{NP} : we' \quad \mathsf{S\backslash NP} : \lambda y.helped'(paint'house')Hans'y \;\Rightarrow_{<}\; \mathsf{S} : helped'(paint'house')Hans'we'$$

$$
\begin{aligned}
\text{we} &\vdash \mathsf{NP} : we' \\
\text{helped} &\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy \\
\text{Hans} &\vdash \mathsf{NP} : Hans' \\
\text{paint} &\vdash \mathsf{VP/NP} : \lambda x.paint'x \\
\text{the house} &\vdash \mathsf{NP} : house'
\end{aligned}
$$

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy$	$\mathsf{NP} : Hans'$	$\mathsf{VP/NP} : \lambda x.paint'x$	$\mathsf{NP} : house'$

$$\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'fHans'y \quad (>)$$

$$\mathsf{VP} : paint'house' \quad (>)$$

$$\mathsf{S\backslash NP} : \lambda y.helped'(paint'house')Hans'y \quad (>)$$

$$\mathsf{S} : helped'(paint'house')Hans'we' \quad (<)$$

we	$\vdash \mathsf{NP} : we'$	$\mathsf{NP} \rightarrow$	we
helped	$\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy$	$\mathsf{S} \rightarrow$	NP helped NP VP
Hans	$\vdash \mathsf{NP} : Hans'$	$\mathsf{NP} \rightarrow$	Hans
paint	$\vdash \mathsf{VP/NP} : \lambda x.paint'x$	VP $\rightarrow$	paint NP
the house	$\vdash \mathsf{NP} : house'$	NP $\rightarrow$	house

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)

we	helped	Hans	paint	the house
$\mathsf{NP} : we'$	$\mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy$	$\mathsf{NP} : Hans'$	$\mathsf{VP/NP} : \lambda x.paint'x$	$\mathsf{NP} : house'$

$$\mathsf{S\backslash NP/VP} : \lambda f.\lambda y.helped'fHans'y \quad (>)$$

$$\mathsf{VP} : paint'house' \quad (>)$$

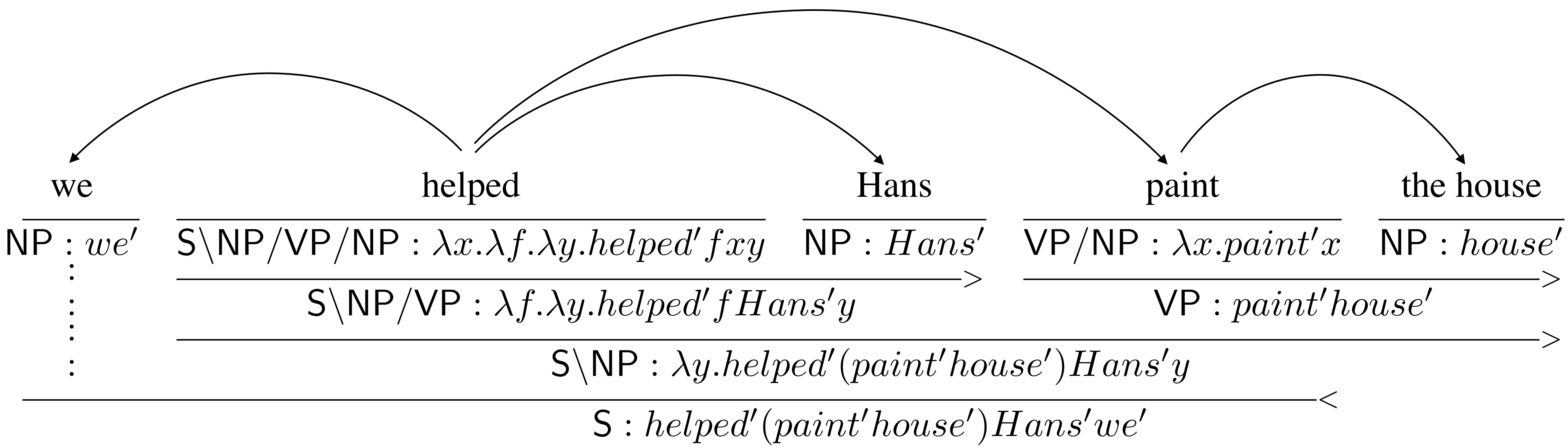
$$\mathsf{S\backslash NP} : \lambda y.helped'(paint'house')Hans'y \quad (>)$$

$$\mathsf{S} : helped'(paint'house')Hans'we' \quad (<)$$

we	$\vdash \mathsf{NP} : we'$	$\mathsf{NP} \rightarrow$	we
helped	$\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'fxy$	$\mathsf{S} \rightarrow$	NP helped NP VP
Hans	$\vdash \mathsf{NP} : Hans'$	$\mathsf{NP} \rightarrow$	Hans
paint	$\vdash \mathsf{VP/NP} : \lambda x.paint'x$	VP $\rightarrow$	paint NP
the house	$\vdash \mathsf{NP} : house'$	NP $\rightarrow$	house

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)

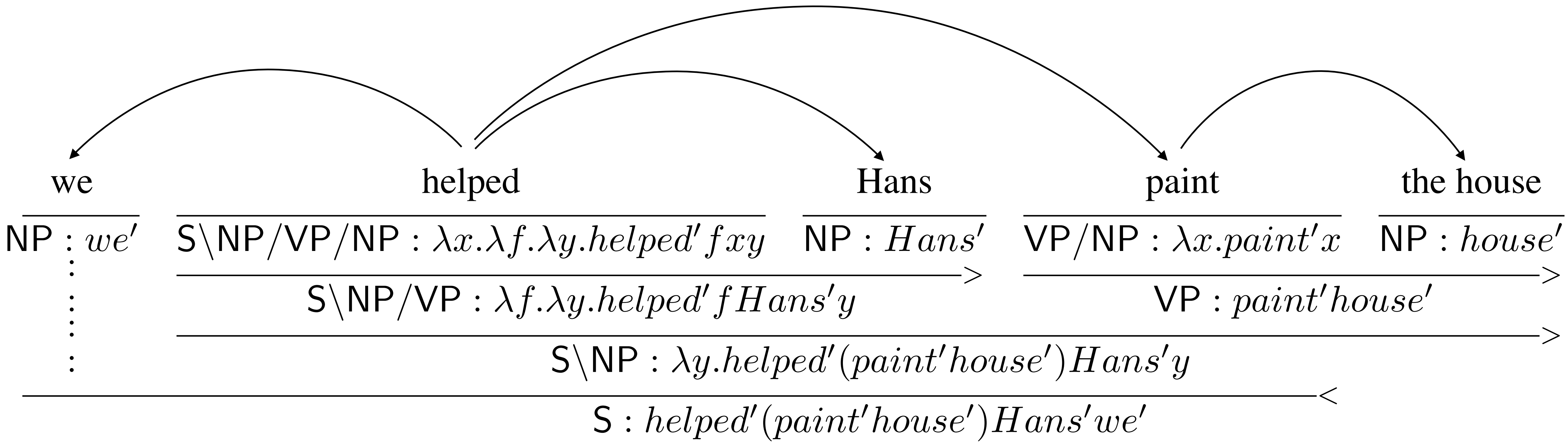


we	$\vdash$ NP : we'	NP $\rightarrow$	we
helped	$\vdash$ S\NP/VP/NP : $\lambda x.\lambda f.\lambda y.helped' f x y$	S $\rightarrow$	NP helped NP VP
Hans	$\vdash$ NP : $Hans'$	NP $\rightarrow$	Hans
paint	$\vdash$ VP/NP : $\lambda x.paint' x$	VP $\rightarrow$	paint NP
the house	$\vdash$ NP : $house'$	NP $\rightarrow$	house

Categorial grammar

CG is context-free (Bar-Hillel et al., 1964)

it is also a projective *dependency grammar* (Hays, 1964; Gaifman, 1965)



we	$\vdash \mathsf{NP} : we'$	$\mathsf{NP} \rightarrow$	we
helped	$\vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,f\,x\,y$	$\mathsf{S} \rightarrow$	NP helped NP VP
Hans	$\vdash \mathsf{NP} : Hans'$	$\mathsf{NP} \rightarrow$	Hans
paint	$\vdash \mathsf{VP/NP} : \lambda x.paint'\,x$	$\mathrm{VP} \rightarrow$	paint NP
the house	$\vdash \mathsf{NP} : house'$	$\mathrm{NP} \rightarrow$	house

Combinatory categorial grammar

mer em Hans es huus hälfed aastriiche

$$\begin{aligned}
\text{mer} &\vdash \mathsf{NP} : we' \\
\text{em Hans} &\vdash \mathsf{NP} : Hans' \\
\text{es huus} &\vdash \mathsf{NP} : house' \\
\text{hälfed} &\vdash \mathsf{S}\backslash\mathsf{NP}\backslash\mathsf{NP}/\mathsf{VP} : \lambda f.\lambda x.\lambda y.helped'fxy \\
\text{aastriiche} &\vdash \mathsf{VP}\backslash\mathsf{NP} : \lambda x.paint'x
\end{aligned}$$

Combinatory categorial grammar

$$\dfrac{\text{mer}}{\mathsf{NP} : we'} \quad \dfrac{\text{em Hans}}{\mathsf{NP} : Hans'} \quad \dfrac{\text{es huus}}{\mathsf{NP} : house'} \quad \dfrac{\text{hälfed}}{\mathsf{S}\backslash\mathsf{NP}\backslash\mathsf{NP}/\mathsf{VP} : \lambda f.\lambda x.\lambda y.helped' fxy} \quad \dfrac{\text{aastriiche}}{\mathsf{VP}\backslash\mathsf{NP} : \lambda x.paint' x}$$

$$\begin{aligned}
\text{mer} &\vdash \mathsf{NP} : we' \\
\text{em Hans} &\vdash \mathsf{NP} : Hans' \\
\text{es huus} &\vdash \mathsf{NP} : house' \\
\text{hälfed} &\vdash \mathsf{S}\backslash\mathsf{NP}\backslash\mathsf{NP}/\mathsf{VP} : \lambda f.\lambda x.\lambda y.helped' fxy \\
\text{aastriiche} &\vdash \mathsf{VP}\backslash\mathsf{NP} : \lambda x.paint' x
\end{aligned}$$

Combinatory categorial grammar

$$\frac{\text{mer}}{\mathsf{NP} : we'} \quad \frac{\text{em Hans}}{\mathsf{NP} : Hans'} \quad \frac{\text{es huus}}{\mathsf{NP} : house'} \quad \frac{\text{hälfed}}{\mathsf{S}\backslash\mathsf{NP}\backslash\mathsf{NP}/\mathsf{VP} : \lambda f.\lambda x.\lambda y.helped'\,fxy} \quad \frac{\text{aastriiche}}{\mathsf{VP}\backslash\mathsf{NP} : \lambda x.paint'\,x}$$

$$\begin{aligned}
\text{mer} &\vdash \mathsf{NP} : we' \\
\text{em Hans} &\vdash \mathsf{NP} : Hans' \\
\text{es huus} &\vdash \mathsf{NP} : house' \\
\text{hälfed} &\vdash \mathsf{S}\backslash\mathsf{NP}\backslash\mathsf{NP}/\mathsf{VP} : \lambda f.\lambda x.\lambda y.helped'\,fxy \\
\text{aastriiche} &\vdash \mathsf{VP}\backslash\mathsf{NP} : \lambda x.paint'\,x
\end{aligned}$$

Combinatory categorial grammar

forward composition (degree n)

$$A/B : f \quad B|_1C_1 ... |_nC_n : \lambda x_n ... \lambda x_1 . g(x_1, ..., x_n)$$
$$\Rightarrow A|_1C_1 ... |_nC_n : \lambda x_n ... \lambda x_1 . f g(x_1, ..., x_n)$$

mer	em Hans	es huus	hälfed	aastriiche
$\mathsf{NP} : we'$	$\mathsf{NP} : Hans'$	$\mathsf{NP} : house'$	$\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' fxy$	$\mathsf{VP\backslash NP} : \lambda x.paint' x$

$$\text{mer} \vdash \mathsf{NP} : we'$$
$$\text{em Hans} \vdash \mathsf{NP} : Hans'$$
$$\text{es huus} \vdash \mathsf{NP} : house'$$
$$\text{hälfed} \vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' fxy$$
$$\text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint' x$$

Combinatory categorial grammar

forward composition (degree n)

$$A/B : f \quad B|_1C_1 \dots |_nC_n : \lambda x_n \dots \lambda x_1 . g(x_1, \dots, x_n)$$
$$\Rightarrow A|_1C_1 \dots |_nC_n : \lambda x_n \dots \lambda x_1 . f g(x_1, \dots, x_n)$$

$$\dfrac{\text{mer}}{\mathsf{NP} : we'} \quad \dfrac{\text{em Hans}}{\mathsf{NP} : Hans'} \quad \dfrac{\text{es huus}}{\mathsf{NP} : house'} \quad \dfrac{\dfrac{\text{hälfed}}{\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'\,fxy} \quad \dfrac{\text{aastriiche}}{\mathsf{VP\backslash NP} : \lambda x.paint'\,x}}{\mathsf{S\backslash NP\backslash NP\backslash NP} : \lambda z.\lambda x.\lambda y.helped'\,(paint'\,z)xy}{>}\mathbf{B}_\times$$

$$\begin{aligned}
\text{mer} &\vdash \mathsf{NP} : we' \\
\text{em Hans} &\vdash \mathsf{NP} : Hans' \\
\text{es huus} &\vdash \mathsf{NP} : house' \\
\text{hälfed} &\vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'\,fxy \\
\text{aastriiche} &\vdash \mathsf{VP\backslash NP} : \lambda x.paint'\,x
\end{aligned}$$

Combinatory categorial grammar

forward composition (degree n)

$$A/B : f \quad B|_1 C_1 ... |_n C_n : \lambda x_n ... \lambda x_1 . g(x_1, ..., x_n)$$
$$\Rightarrow A|_1 C_1 ... |_n C_n : \lambda x_n ... \lambda x_1 . f g(x_1, ..., x_n)$$

$$\dfrac{\text{mer}}{\mathsf{NP} : we'} \quad \dfrac{\text{em Hans}}{\mathsf{NP} : Hans'} \quad \dfrac{\text{es huus}}{\mathsf{NP} : house'} \quad \dfrac{\dfrac{\text{hälfed}}{\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' fxy} \quad \dfrac{\text{aastriiche}}{\mathsf{VP\backslash NP} : \lambda x.paint' x}}{\mathsf{S\backslash NP\backslash NP\backslash NP} : \lambda z.\lambda x.\lambda y.helped'(paint' z)xy}{>}\mathbf{B}_\times$$

$$\text{mer} \vdash \mathsf{NP} : we'$$
$$\text{em Hans} \vdash \mathsf{NP} : Hans'$$
$$\text{es huus} \vdash \mathsf{NP} : house'$$
$$\text{hälfed} \vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' fxy$$
$$\text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint' x$$

Combinatory categorial grammar

forward composition (degree n)

$$A/B : f \quad B|_1C_1...|_nC_n : \lambda x_n...\lambda x_1.g(x_1, ..., x_n)$$
$$\Rightarrow A|_1C_1...|_nC_n : \lambda x_n...\lambda x_1.fg(x_1, ..., x_n)$$

mer	em Hans	es huus	hälfed	aastriiche
$\mathsf{NP} : we'$	$\mathsf{NP} : Hans'$	$\mathsf{NP} : house'$	$\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'fxy$	$\mathsf{VP\backslash NP} : \lambda x.paint'x$

$$\frac{\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'fxy \quad \mathsf{VP\backslash NP} : \lambda x.paint'x}{\mathsf{S\backslash NP\backslash NP\backslash NP} : \lambda z.\lambda x.\lambda y.helped'(paint'z)xy} >\mathbf{B}_\times$$

$$\begin{aligned}
\text{mer} &\vdash \mathsf{NP} : we' \\
\text{em Hans} &\vdash \mathsf{NP} : Hans' \\
\text{es huus} &\vdash \mathsf{NP} : house' \\
\text{hälfed} &\vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'fxy \\
\text{aastriiche} &\vdash \mathsf{VP\backslash NP} : \lambda x.paint'x
\end{aligned}$$

Combinatory categorial grammar

forward composition (degree n)

$$A/B : f \quad B|_1 C_1 ... |_n C_n : \lambda x_n ... \lambda x_1 . g(x_1, ..., x_n)$$
$$\Rightarrow A|_1 C_1 ... |_n C_n : \lambda x_n ... \lambda x_1 . f g(x_1, ..., x_n)$$

$$\dfrac{\text{mer}}{\mathsf{NP} : we'} \quad \dfrac{\text{em Hans}}{\mathsf{NP} : Hans'} \quad \dfrac{\text{es huus}}{\mathsf{NP} : house'} \quad \cfrac{\dfrac{\text{hälfed}}{\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' fxy} \quad \dfrac{\text{aastriiche}}{\mathsf{VP\backslash NP} : \lambda x.paint' x}}{\mathsf{S\backslash NP\backslash NP\backslash NP} : \lambda z.\lambda x.\lambda y.helped'(paint' z)xy} {>}\mathbf{B}_\times$$

$$\begin{aligned}
\text{mer} &\vdash \mathsf{NP} : we' \\
\text{em Hans} &\vdash \mathsf{NP} : Hans' \\
\text{es huus} &\vdash \mathsf{NP} : house' \\
\text{hälfed} &\vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' fxy \\
\text{aastriiche} &\vdash \mathsf{VP\backslash NP} : \lambda x.paint' x
\end{aligned}$$

Combinatory categorial grammar

forward composition (degree n)

$$A/B : f \quad B|_1 C_1 ... |_n C_n : \lambda x_n ... \lambda x_1 . g(x_1, ..., x_n)$$
$$\Rightarrow A|_1 C_1 ... |_n C_n : \lambda x_n ... \lambda x_1 . f g(x_1, ..., x_n)$$

mer	em Hans	es huus	hälfed	aastriiche
$\mathsf{NP} : we'$	$\mathsf{NP} : Hans'$	$\mathsf{NP} : house'$	$\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' f x y$	$\mathsf{VP\backslash NP} : \lambda x.paint' x$

$$\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' f x y \quad \mathsf{VP\backslash NP} : \lambda x.paint' x \;\; >\mathbf{B}_\times$$
$$\mathsf{S\backslash NP\backslash NP\backslash NP} : \lambda z.\lambda x.\lambda y.helped'(paint' z) x y$$
$$<\quad \mathsf{S\backslash NP\backslash NP} : \lambda x.\lambda y.helped'(paint' house') x y$$
$$<\quad \mathsf{S\backslash NP} : \lambda y.helped'(paint' house') Hans' y$$
$$<\quad \mathsf{S} : helped'(paint' house') Hans' we'$$

mer $\vdash \mathsf{NP} : we'$

em Hans $\vdash \mathsf{NP} : Hans'$

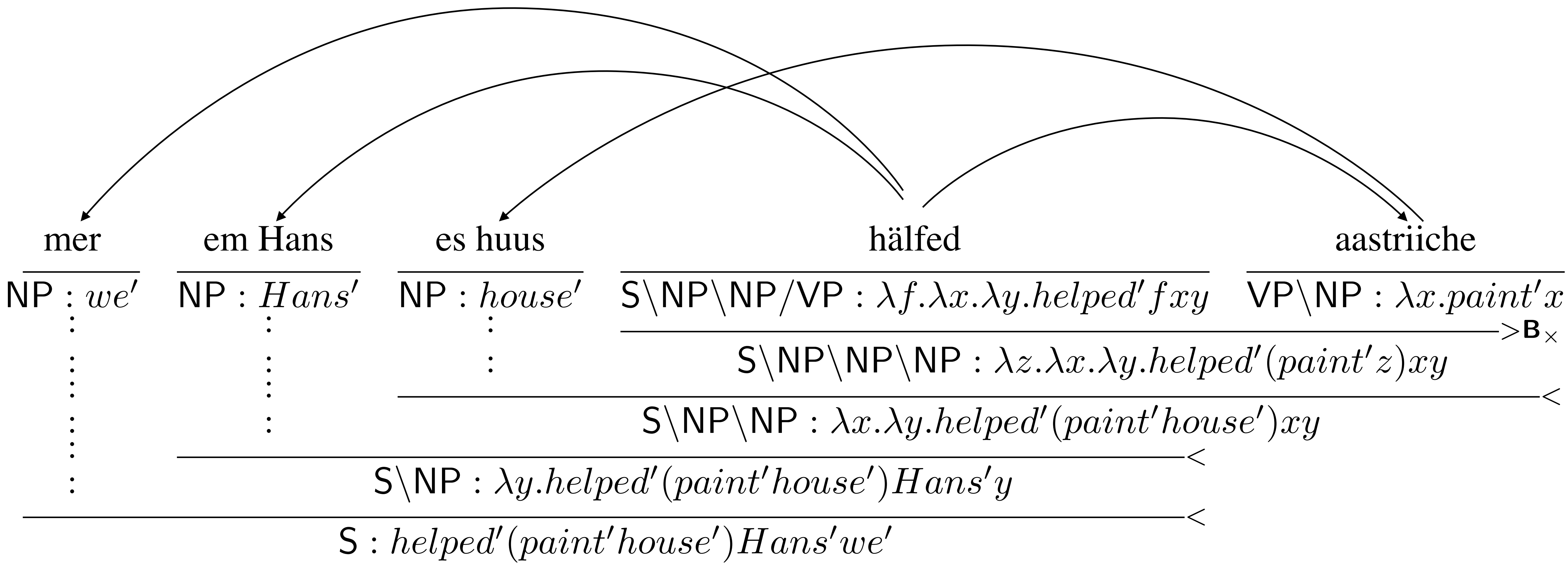
es huus $\vdash \mathsf{NP} : house'$

hälfed $\vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' f x y$

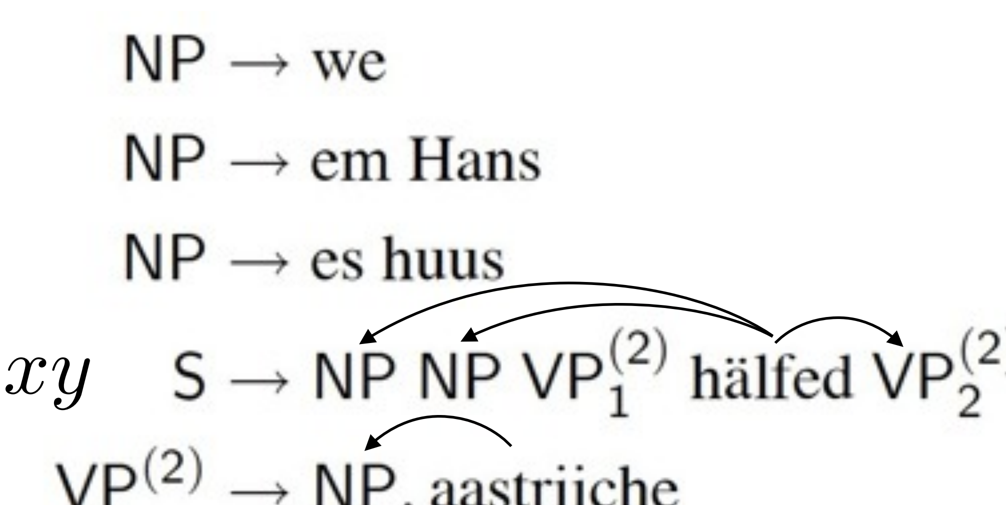
aastriiche $\vdash \mathsf{VP\backslash NP} : \lambda x.paint' x$

Combinatory categorial grammar

is mildly non-projective dependency grammar (Kuhlmann, 2013)

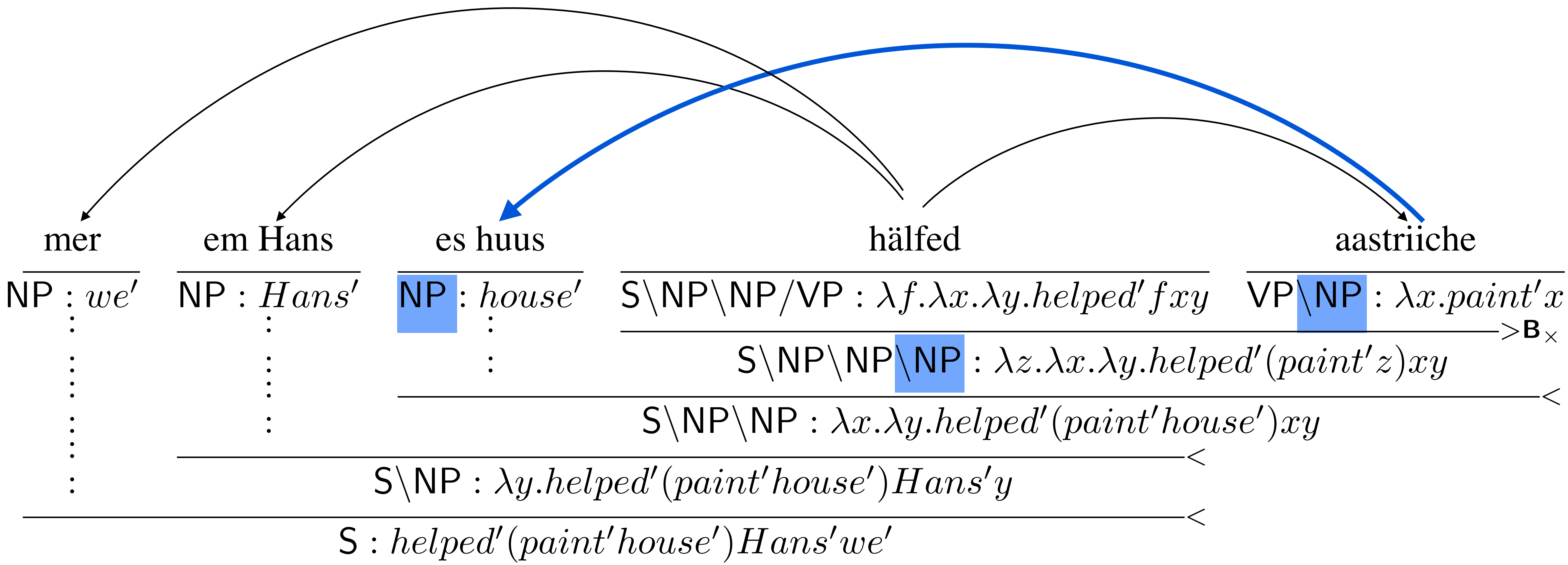


$$
\begin{aligned}
\text{mer} &\vdash \mathsf{NP} : we' \\
\text{em Hans} &\vdash \mathsf{NP} : Hans' \\
\text{es huus} &\vdash \mathsf{NP} : house' \\
\text{hälfed} &\vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' fxy \\
\text{aastriiche} &\vdash \mathsf{VP\backslash NP} : \lambda x.paint' x
\end{aligned}
$$

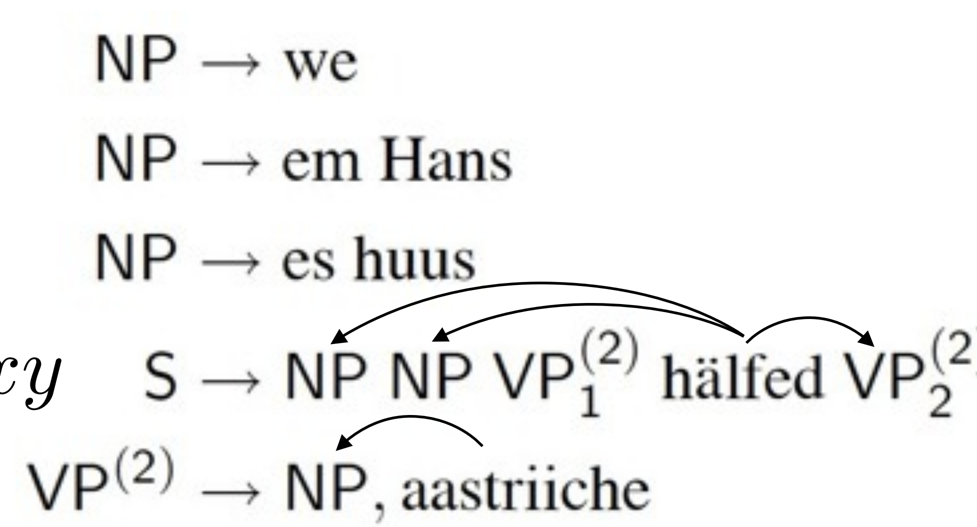


Combinatory categorial grammar

is mildly non-projective dependency grammar (Kuhlmann, 2013)

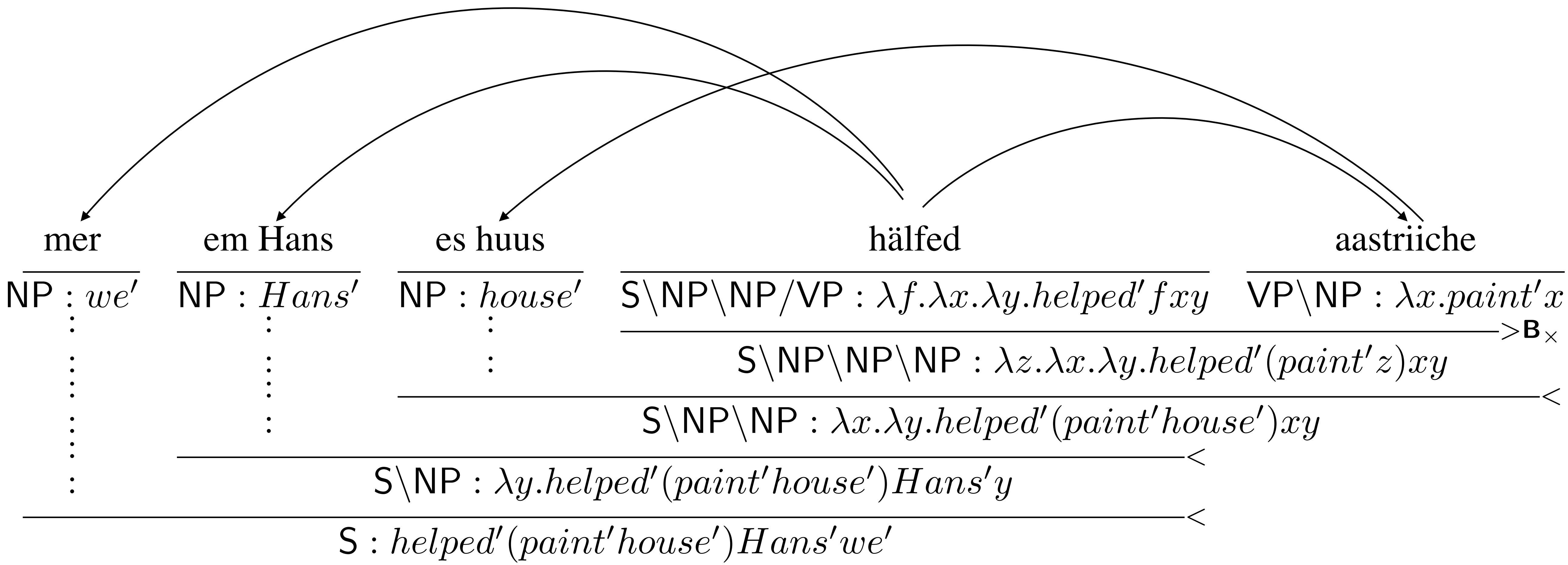


$$\text{mer} \vdash \mathsf{NP} : we'$$
$$\text{em Hans} \vdash \mathsf{NP} : Hans'$$
$$\text{es huus} \vdash \mathsf{NP} : house'$$
$$\text{hälfed} \vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'\,f\,x\,y$$
$$\text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint'\,x$$



Combinatory categorial grammar

is mildly non-projective dependency grammar (Kuhlmann, 2013)



CCG is not context-free

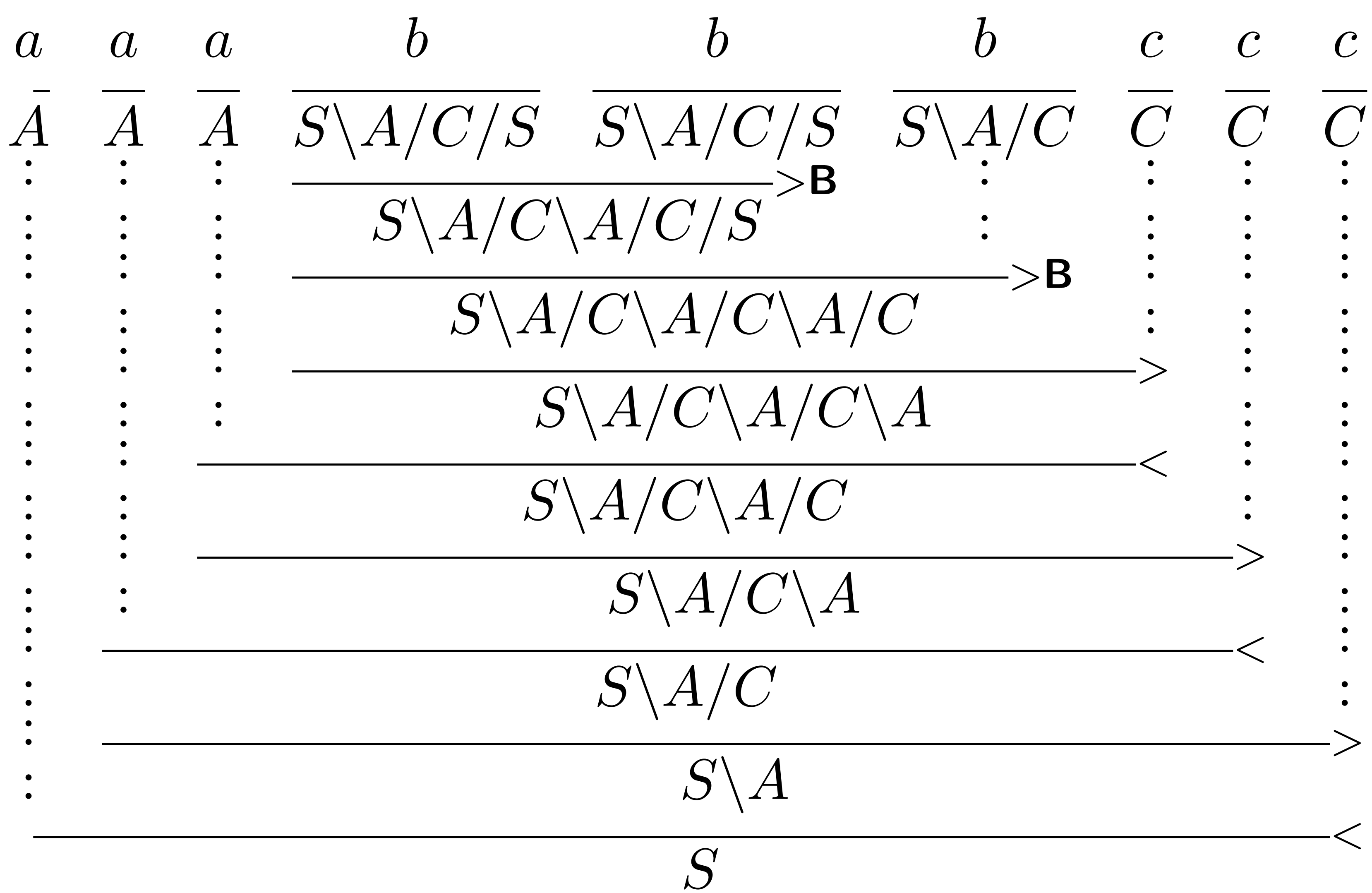
$a \vdash A$

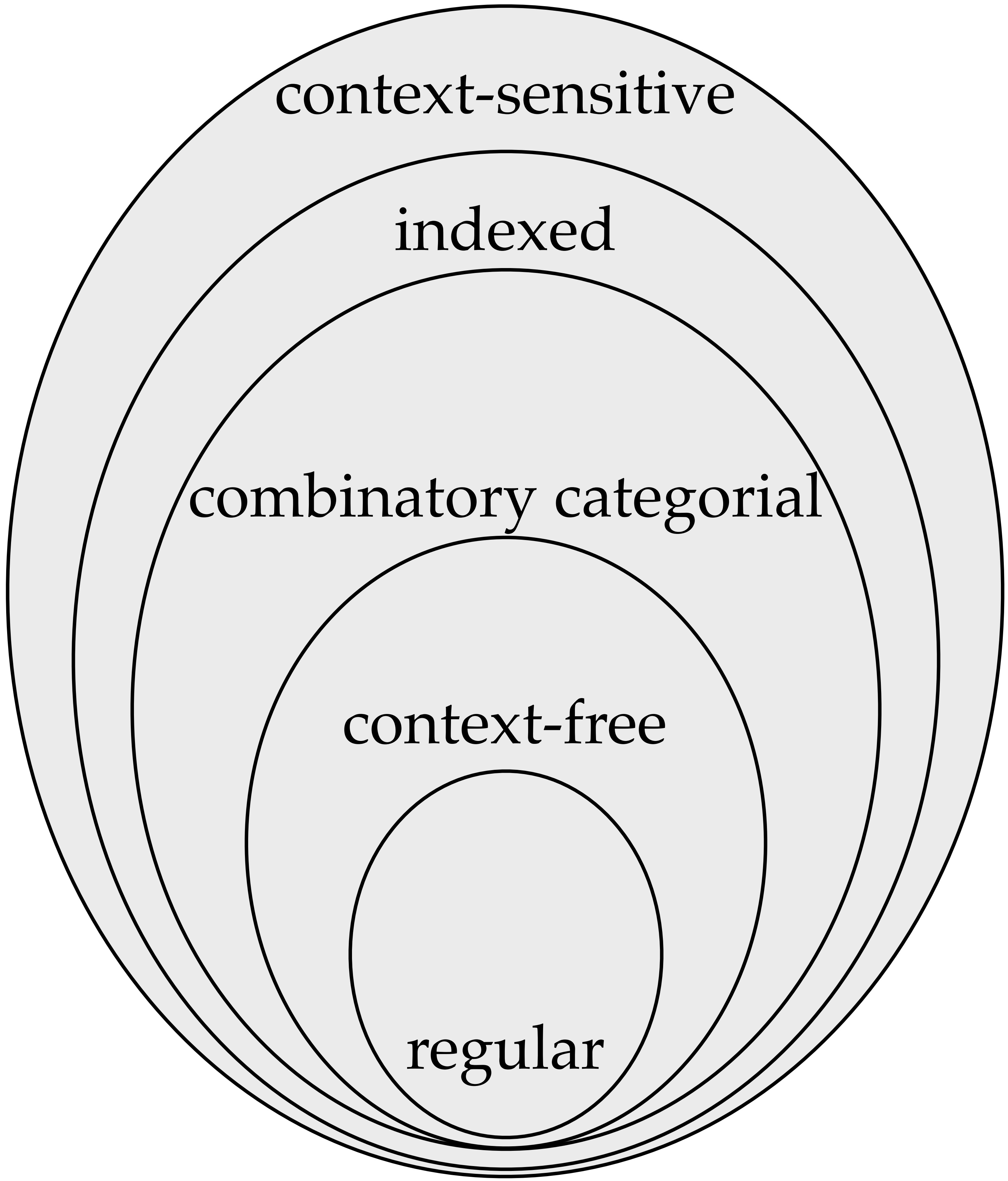
$b \vdash S\backslash A/C$

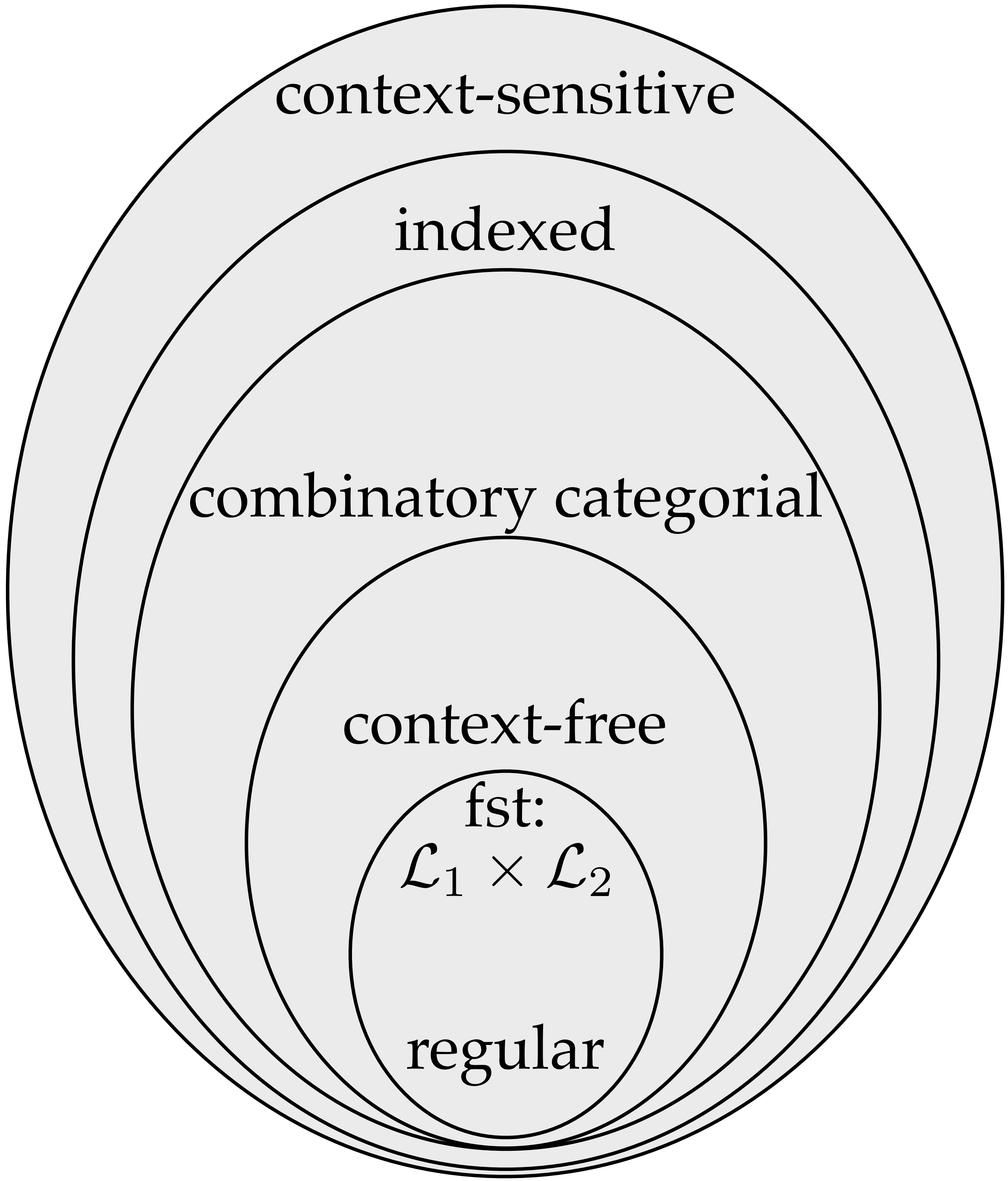
$b \vdash S\backslash A/C/S$

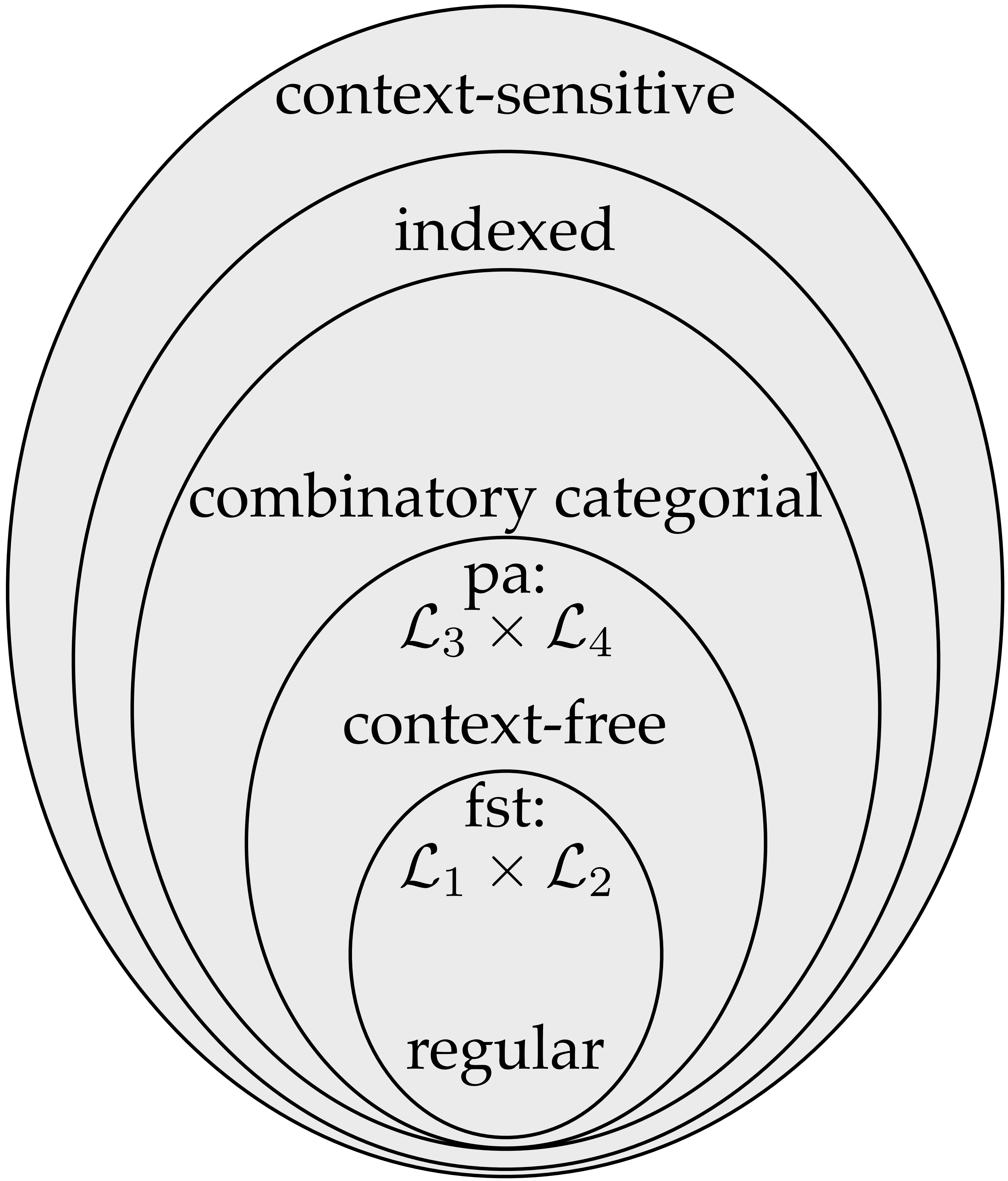
$c \vdash C$

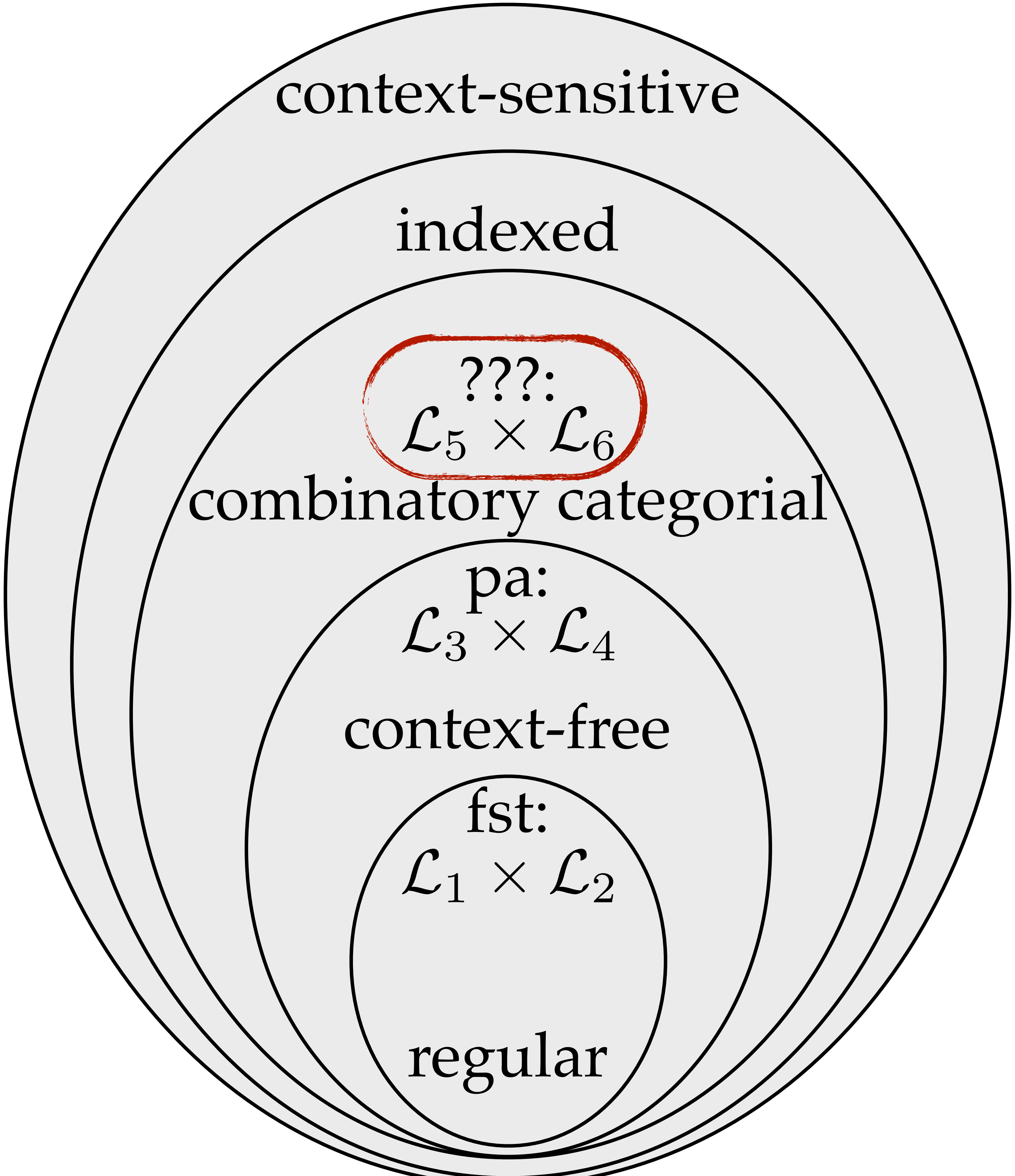
intersected with $a^*b^*c^* \quad = \quad a^nb^nc^n$



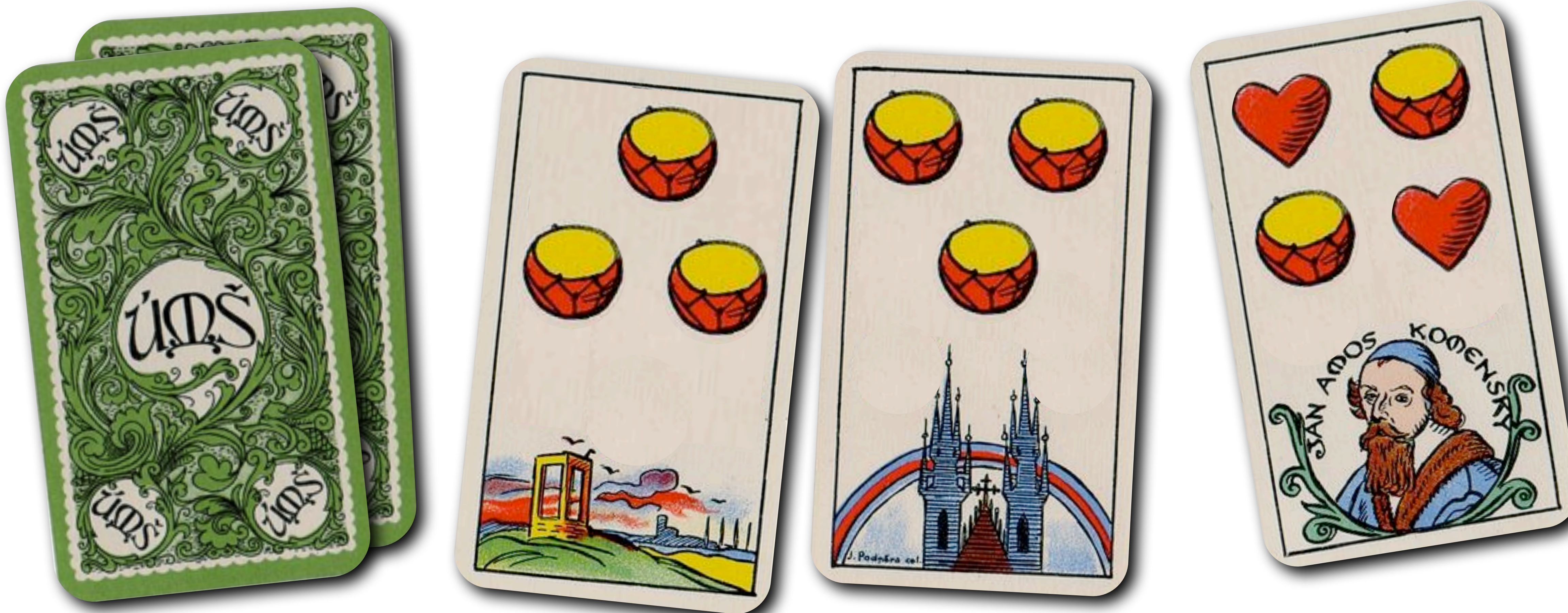

context-sensitive
indexed
combinatory categorial
context-free
regular


context-sensitive
indexed
combinatory categorial
context-free
fst:
$\mathcal{L}_1 \times \mathcal{L}_2$
regular


context-sensitive
indexed
combinatory categorial
pa:
$\mathcal{L}_3 \times \mathcal{L}_4$
context-free
fst:
$\mathcal{L}_1 \times \mathcal{L}_2$
regular


context-sensitive
indexed
???:
$\mathcal{L}_5 \times \mathcal{L}_6$
combinatory categorial
pa:
$\mathcal{L}_3 \times \mathcal{L}_4$
context-free
fst:
$\mathcal{L}_1 \times \mathcal{L}_2$
regular


ÚMŠ


ÚMŠ
JAN AMOS KOMENSKÝ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JAN AMOS KOMENSKY


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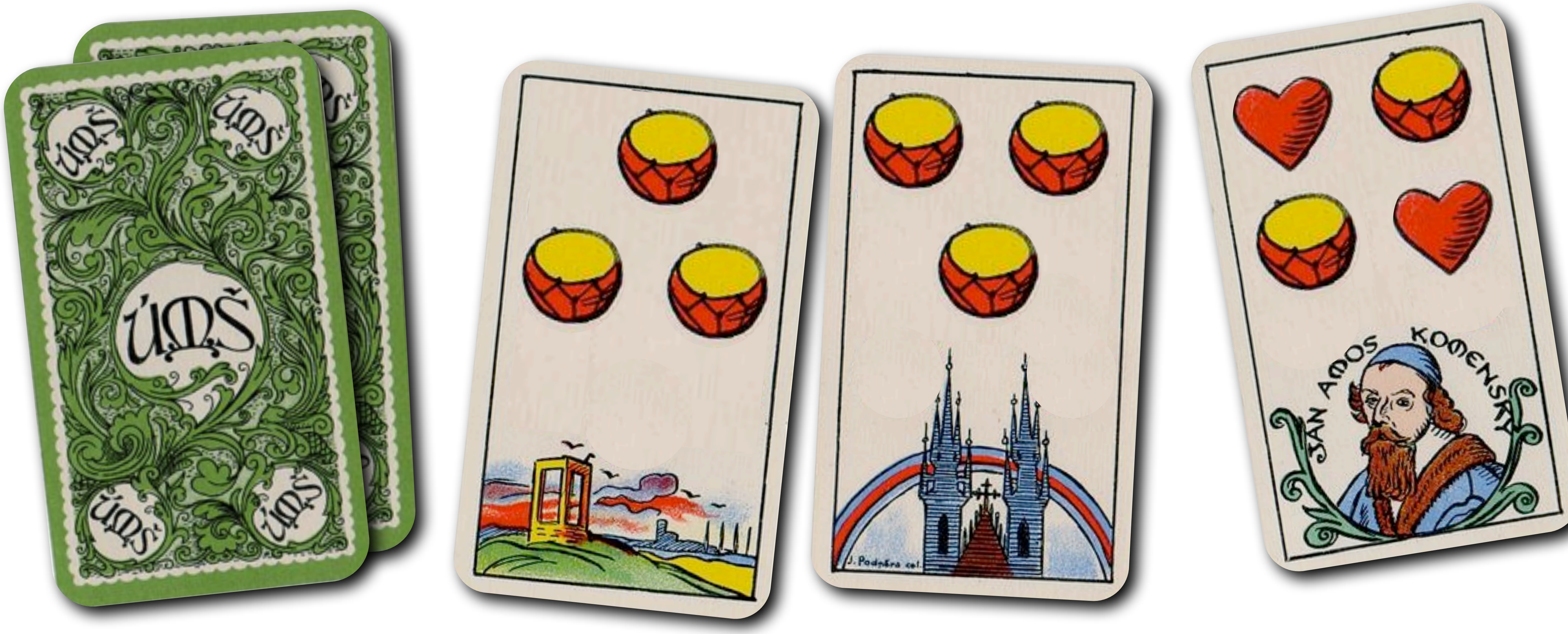
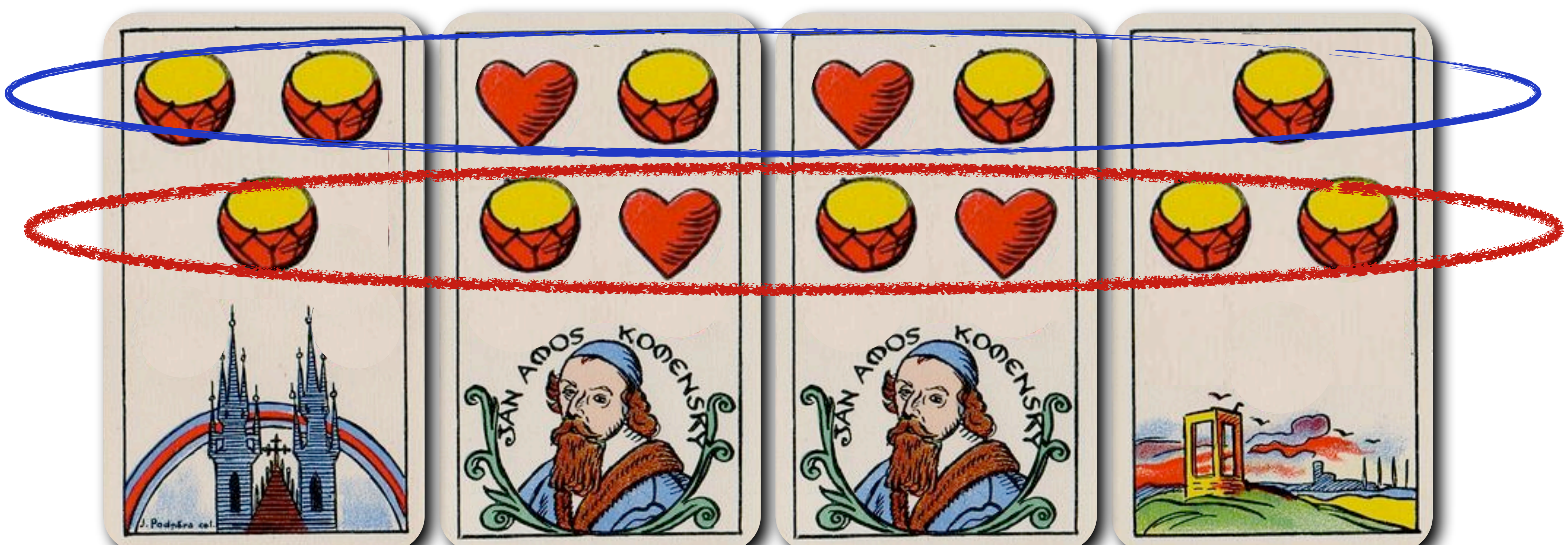
JAN AMOS KOMENSKÝ



Post correspondence problem



$$1 : (bb, b) \quad 2 : (b, bb) \quad 3 : (ab, ba)$$

$$1 : (bb, b) \;\; 2 : (b, bb) \;\; 3 : (ab, ba)$$

Grammar A

$w \vdash S/R/(S/R) : \lambda f.\lambda x.b(b(f(1(x))))$

$w \vdash S/(S/R) : \lambda f.b(b(f(1)))$

$w \vdash S/R : \lambda x.b(b(1(x)))$

$w \vdash S/R/(S/R) : \lambda f.\lambda x.b(f(2(x)))$

$w \vdash S/(S/R) : \lambda f.b(f(2))$

$w \vdash S/R : \lambda x.b(2(x))$

$w \vdash S/R/(S/R) : \lambda f.\lambda x.a(b(f(3(x))))$

$w \vdash S/(S/R) : \lambda f.a(b(f(3)))$

$w \vdash S/R : \lambda x.a(b(3(x)))$

Grammar B

$w \vdash S/R/(S/R) : \lambda f.\lambda x.b(f(1(x)))$

$w \vdash S/(S/R) : \lambda f.b(f(1))$

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$w \vdash S/(S/R) : \lambda f.b(b(f(2)))$

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$$w \vdash S/R/(S/R) : \lambda f.\lambda x.a(b(f(3(x))))$$
$$w \vdash S/(S/R) : \lambda f.a(b(f(3)))$$
$$w \vdash S/R : \lambda x.a(b(3(x)))$$

Grammar B

$$w \vdash S/R/(S/R) : \lambda f.\lambda x.b(f(1(x)))$$
$$w \vdash S/(S/R) : \lambda f.b(f(1))$$
$$w \vdash S/R : \lambda x.b(1(x))$$
$$w \vdash S/R/(S/R) : \lambda f.\lambda x.b(b(f(2(x))))$$
$$w \vdash S/(S/R) : \lambda f.b(b(f(2)))$$
$$w \vdash S/R : \lambda x.b(b(2(x)))$$
$$w \vdash S/R/(S/R) : \lambda f.\lambda x.b(a(f(3(x))))$$
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Grammar B

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Grammar B

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$w \vdash S/(S/R) : \lambda f.b(f(1))$

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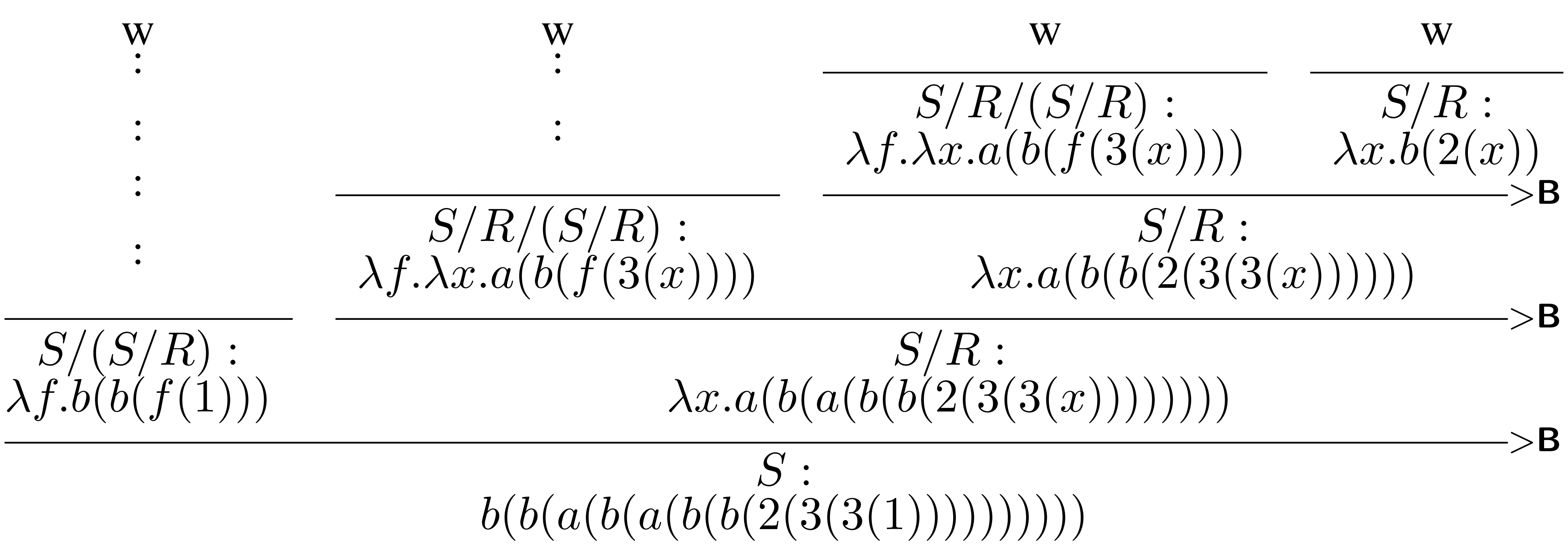

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Grammar B

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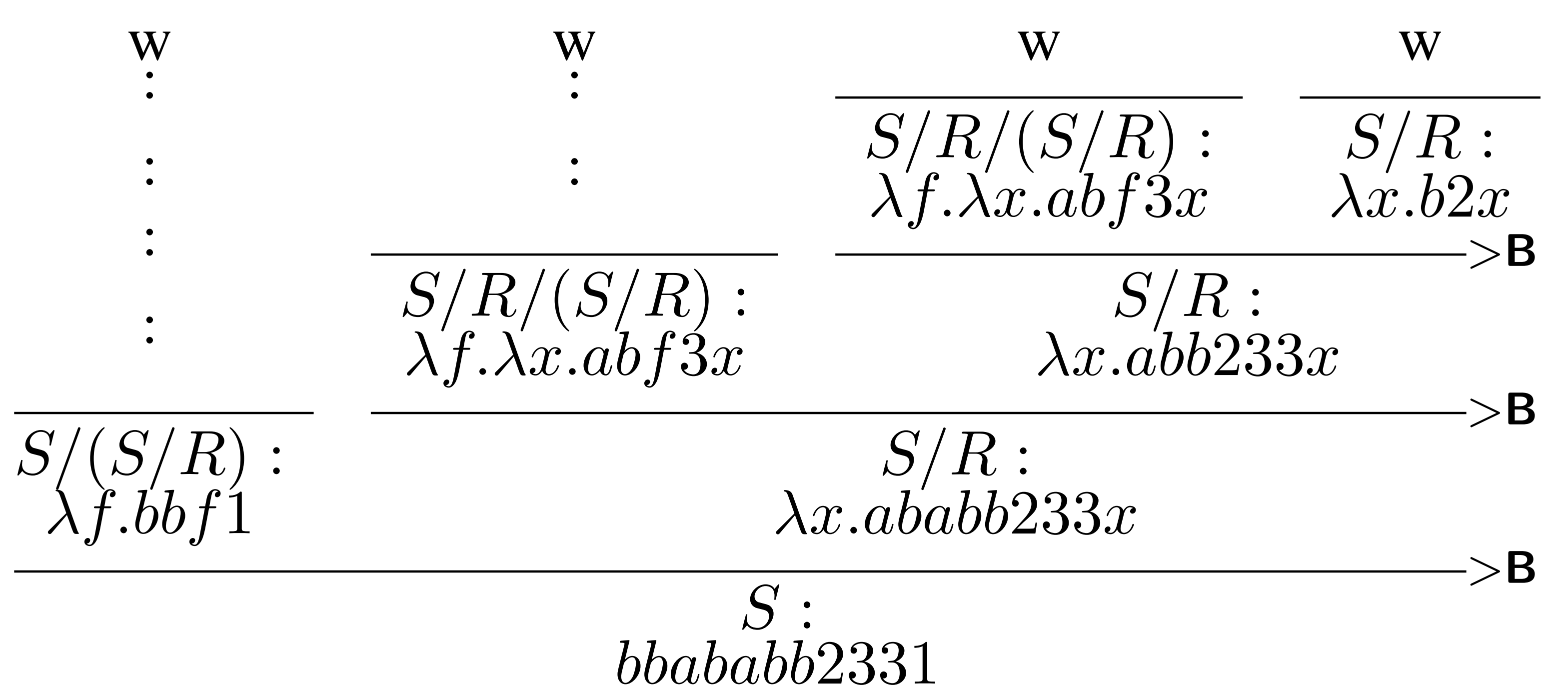

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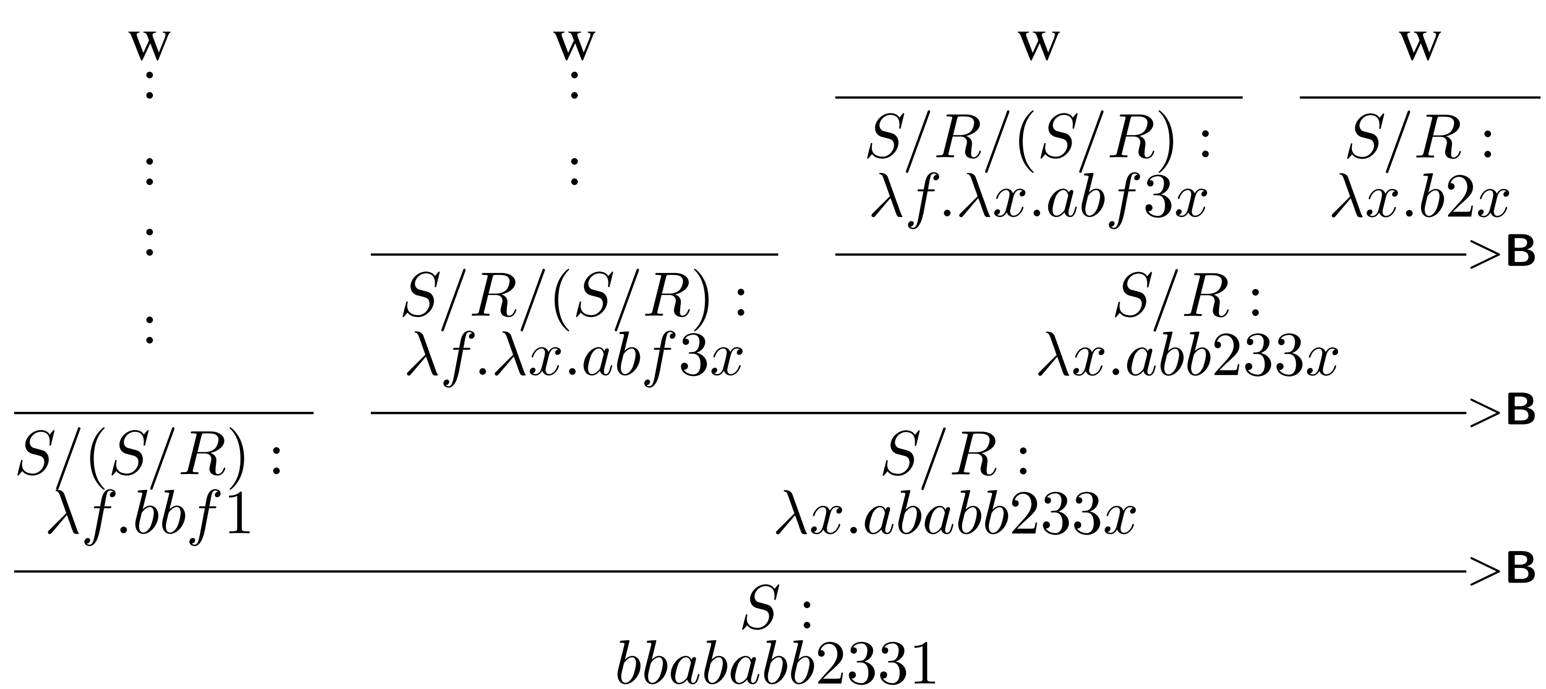
$w \vdash S/R : \lambda x.b(a(3(x)))$

Given two arbitrary CCGs, it is undecidable whether they produce a pair of strings with equivalent semantics.



Desiderata for a formal model of translation

- ☐ Linguistically expressive.
- ☐ Explicit preservation of semantics.
- ☐ Efficient algorithms.
- ☐ Existence of synchronous formalism.

Desiderata for a formal model of translation

- ✔ Linguistically expressive.
- ✔ Explicit preservation of semantics.
- ✔ Efficient algorithms.
- ✘ Existence of synchronous formalism.

Combinatory Categorial Grammar

Desiderata for a formal model of translation

- ☑ Linguistically expressive.
- ☑ Explicit preservation of semantics.
- ☑ Efficient algorithms.
- ☑ Existence of synchronous formalism.

Synchronous Combinatory Categorial Grammar

Desiderata for a formal model of translation

- ☑ Linguistically expressive.
- ☑ Explicit preservation of semantics.
- ☑ Efficient algorithms.
- ☑ Existence of synchronous formalism.

Synchronous Combinatory Categorial Grammar

Fine print for rest of this talk

- **No** rule restrictions (*pure* CCG).
- **No** higher-order argument categories.
- **No** type-raising, substitution, or D combinator.
- Bound variables appear exactly once in a term.

Synchronous CCG

$$\begin{bmatrix} \text{mer} \vdash \mathsf{NP} : we' \\ \text{we} \vdash \mathsf{NP} : we' \end{bmatrix}$$

$$\begin{bmatrix} \text{em Hans} \vdash \mathsf{NP} : Hans' \\ \text{Hans} \vdash \mathsf{NP} : Hans' \end{bmatrix}$$

$$\begin{bmatrix} \text{es huus} \vdash \mathsf{NP} : house' \\ \text{the house} \vdash \mathsf{NP} : house' \end{bmatrix}$$

$$\begin{bmatrix} \text{hälfed} \vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped' fxy \\ \text{helped} \vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped' fxy \end{bmatrix}$$

$$\begin{bmatrix} \text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint' x \\ \text{paint} \vdash \mathsf{VP/NP} : \lambda x.paint' x \end{bmatrix}$$

Synchronous CCG

Left projection

$$\begin{bmatrix} \text{mer} \vdash \mathsf{NP} : we' \\ \text{we} \vdash \mathsf{NP} : we' \end{bmatrix}$$

$$\begin{bmatrix} \text{em Hans} \vdash \mathsf{NP} : Hans' \\ \text{Hans} \vdash \mathsf{NP} : Hans' \end{bmatrix}$$

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$$\begin{bmatrix} \text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint'x \\ \text{paint} \vdash \mathsf{VP/NP} : \lambda x.paint'x \end{bmatrix}$$

Synchronous CCG

$$\begin{bmatrix} \text{mer} \vdash \mathsf{NP} : we' \\ \text{we} \vdash \mathsf{NP} : we' \end{bmatrix}$$

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$$\begin{bmatrix} \text{es huus} \vdash \mathsf{NP} : house' \\ \text{the house} \vdash \mathsf{NP} : house' \end{bmatrix}$$

$$\begin{bmatrix} \text{hälfed} \vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'\,fxy \\ \text{helped} \vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy \end{bmatrix}$$

$$\begin{bmatrix} \text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint'x \\ \text{paint} \vdash \mathsf{VP/NP} : \lambda x.paint'x \end{bmatrix}$$

Right projection

Synchronous CCG

Both left and right projections are CCGs.

$$\begin{bmatrix} \text{mer} \vdash \mathsf{NP} : we' \\ \text{we} \vdash \mathsf{NP} : we' \end{bmatrix}$$

$$\begin{bmatrix} \text{em Hans} \vdash \mathsf{NP} : Hans' \\ \text{Hans} \vdash \mathsf{NP} : Hans' \end{bmatrix}$$

$$\begin{bmatrix} \text{es huus} \vdash \mathsf{NP} : house' \\ \text{the house} \vdash \mathsf{NP} : house' \end{bmatrix}$$

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$$\begin{bmatrix} \text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint' x \\ \text{paint} \vdash \mathsf{VP/NP} : \lambda x.paint' x \end{bmatrix}$$

Synchronous CCG

$$\left[\begin{array}{r} \text{mer} \vdash \mathsf{NP} : we' \\ \text{we} \vdash \mathsf{NP} : we' \end{array}\right]$$

$$\left[\begin{array}{r} \text{em Hans} \vdash \mathsf{NP} : Hans' \\ \text{Hans} \vdash \mathsf{NP} : Hans' \end{array}\right]$$

$$\left[\begin{array}{r} \text{es huus} \vdash \mathsf{NP} : house' \\ \text{the house} \vdash \mathsf{NP} : house' \end{array}\right]$$

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Isomorphic up to ordering of bound variables...

Synchronous CCG

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$$\begin{bmatrix} \text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint'x \\ \text{paint} \vdash \mathsf{VP/NP} : \lambda x.paint'x \end{bmatrix}$$

Isomorphic up to ordering of bound variables...

... hence isomorphic up to order and directionality of arguments

Synchronous CCG

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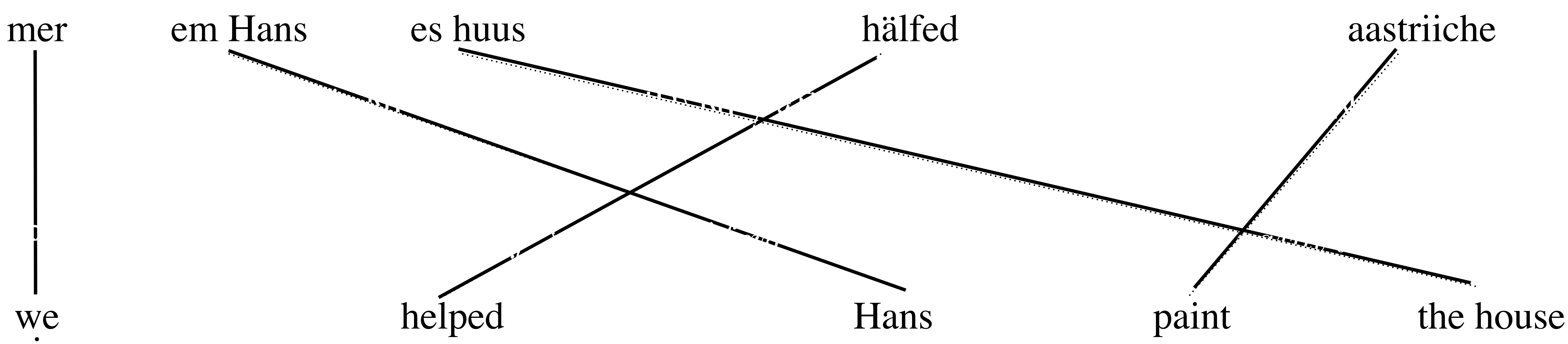
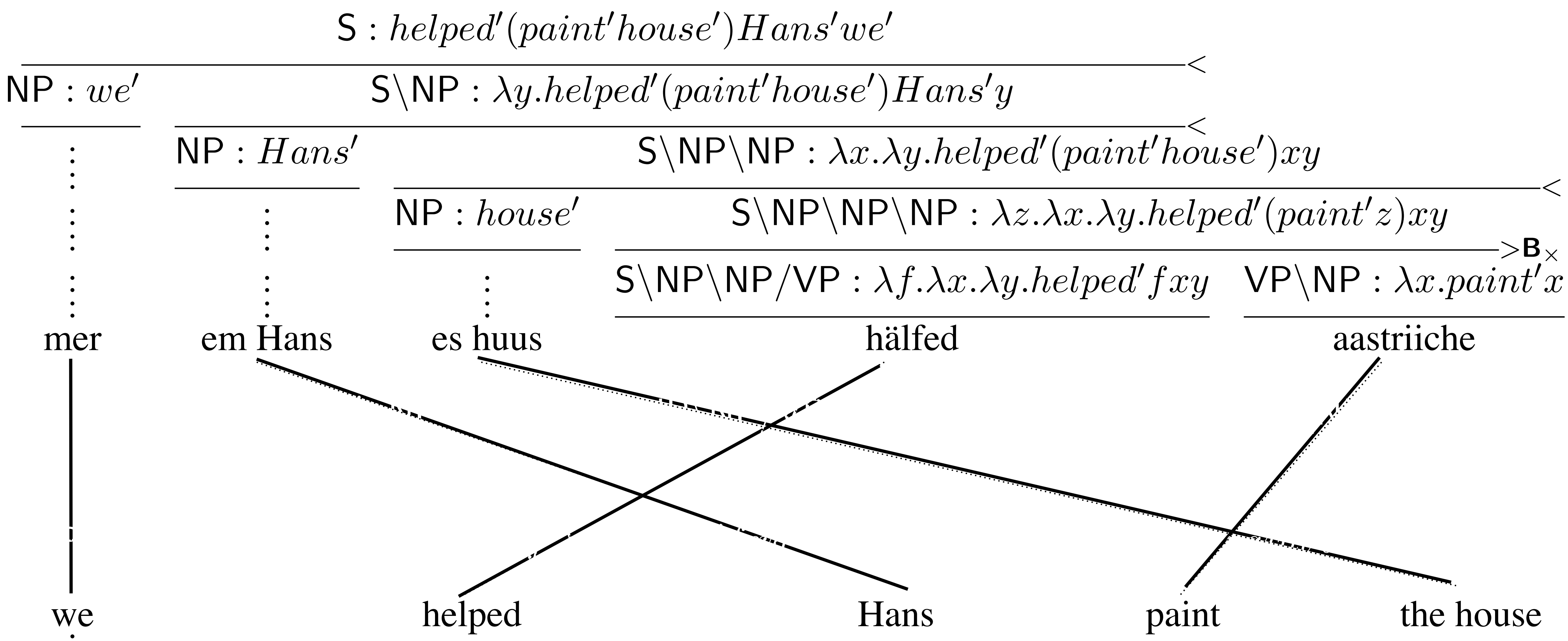
Synchronous CCG

1. Each string must be a permutation, in its projection, of a shared set of lexical entries.



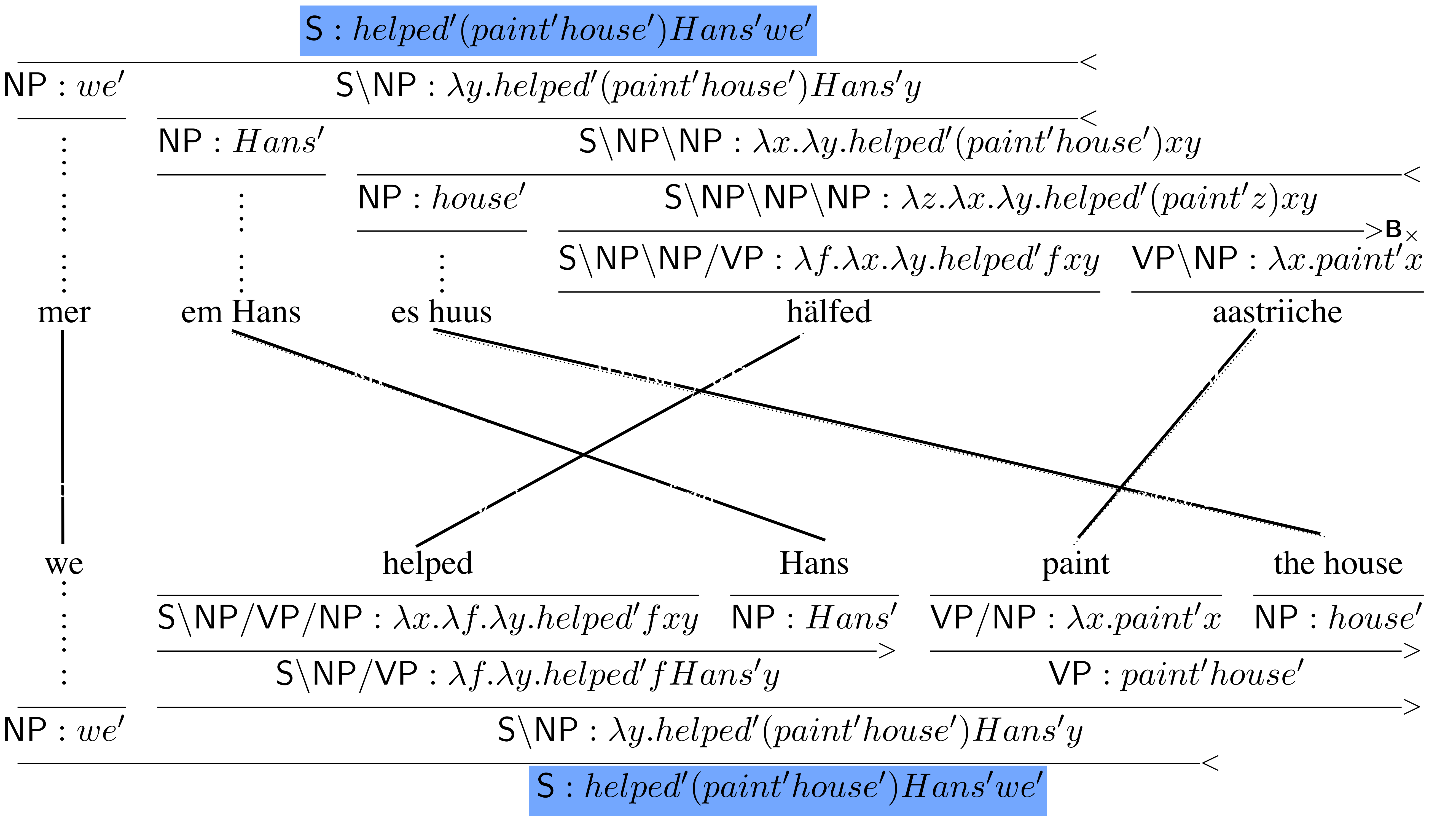
Synchronous CCG



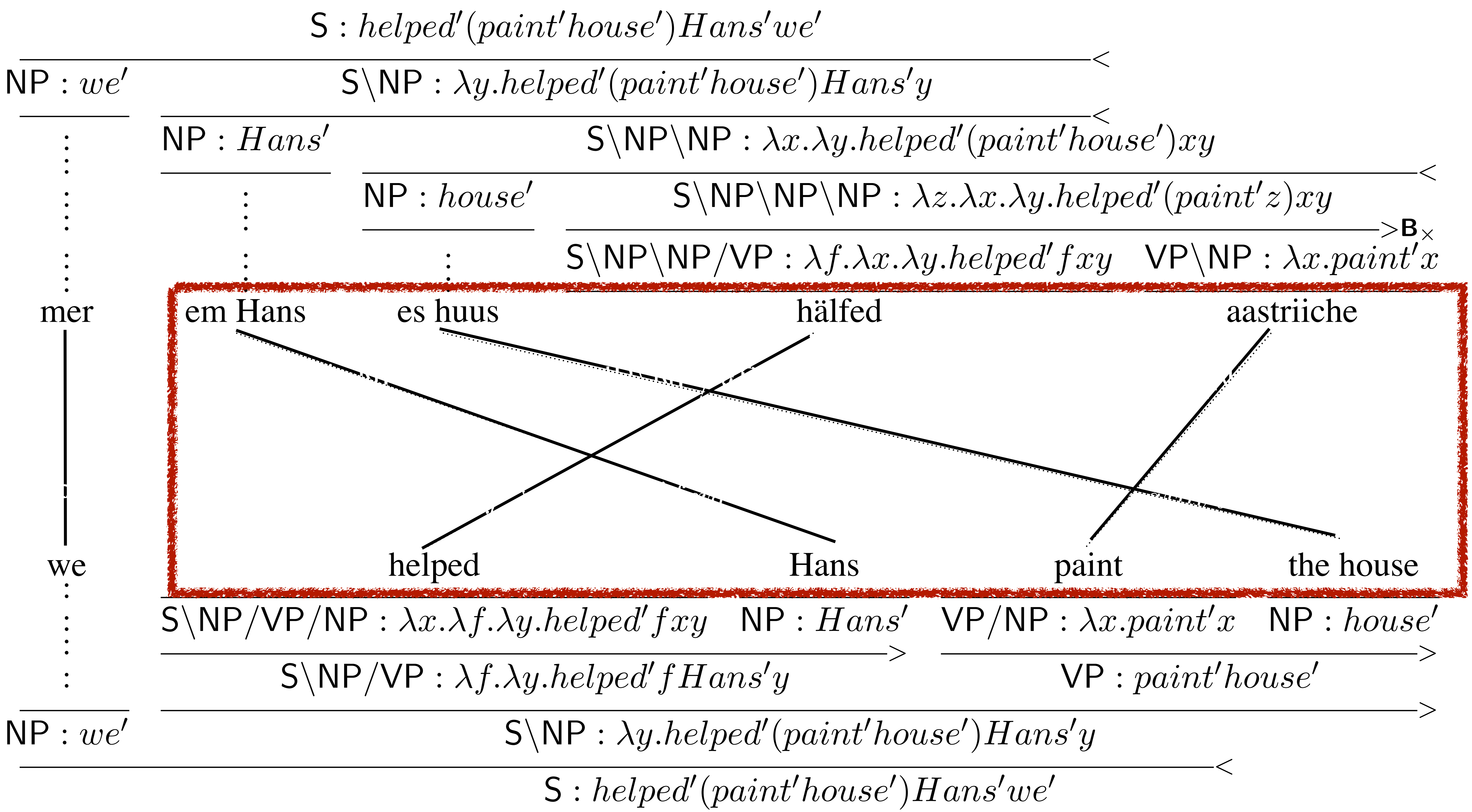
2. Each string must be derivable in its projection.

Synchronous CCG

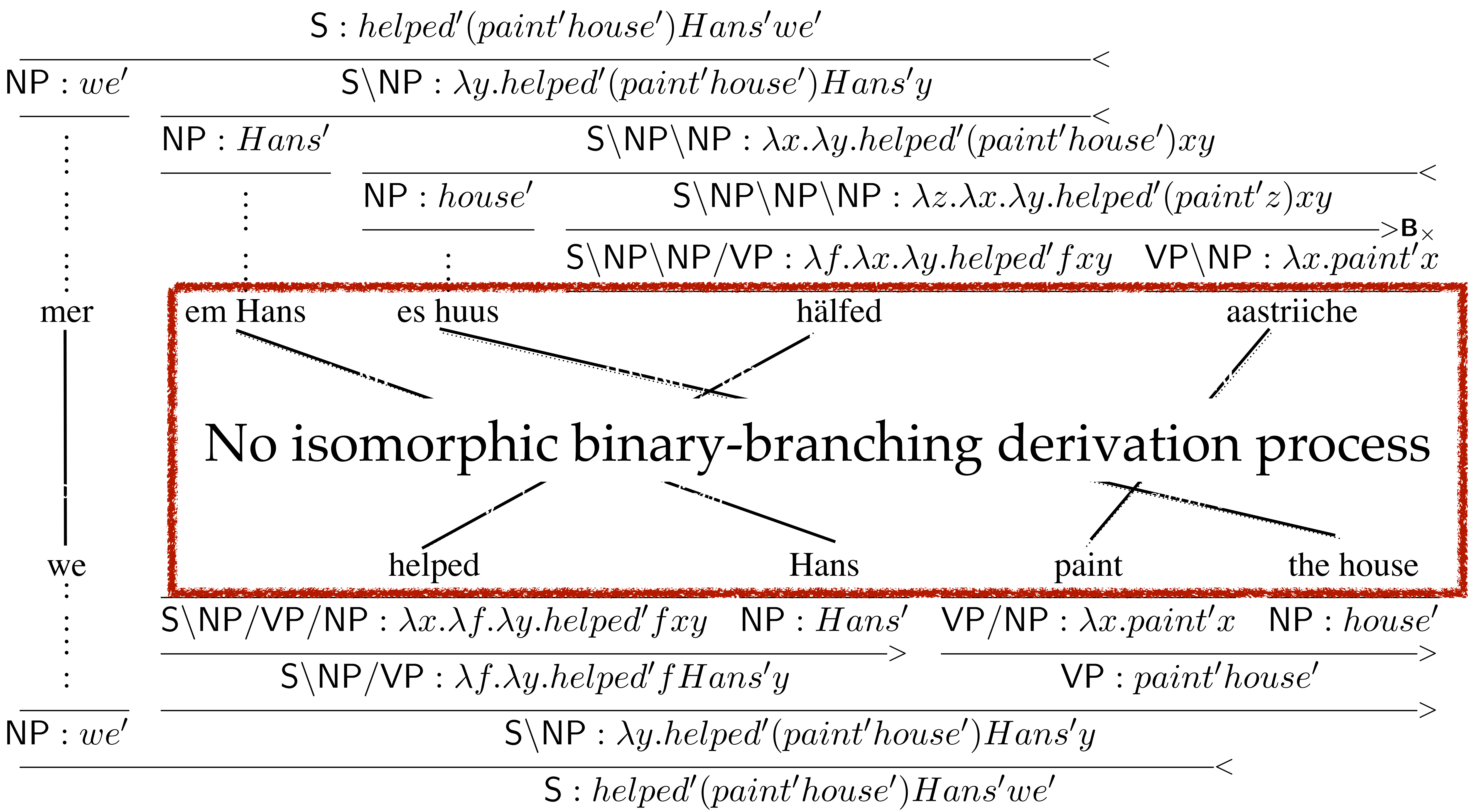
3. There must be a pair of derivations with identical semantics.



Synchronous CCG



Synchronous CCG



Set-theoretic view

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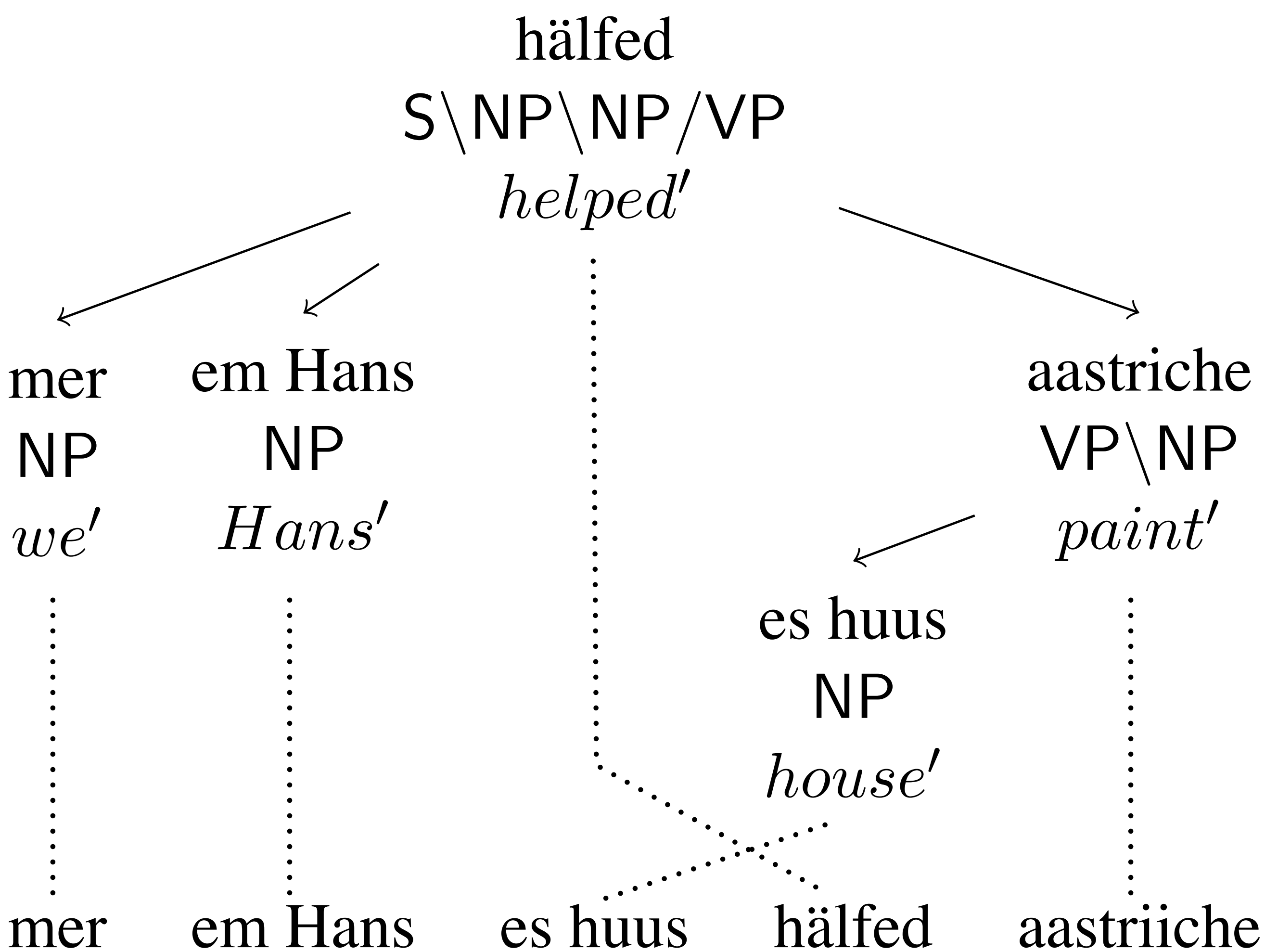
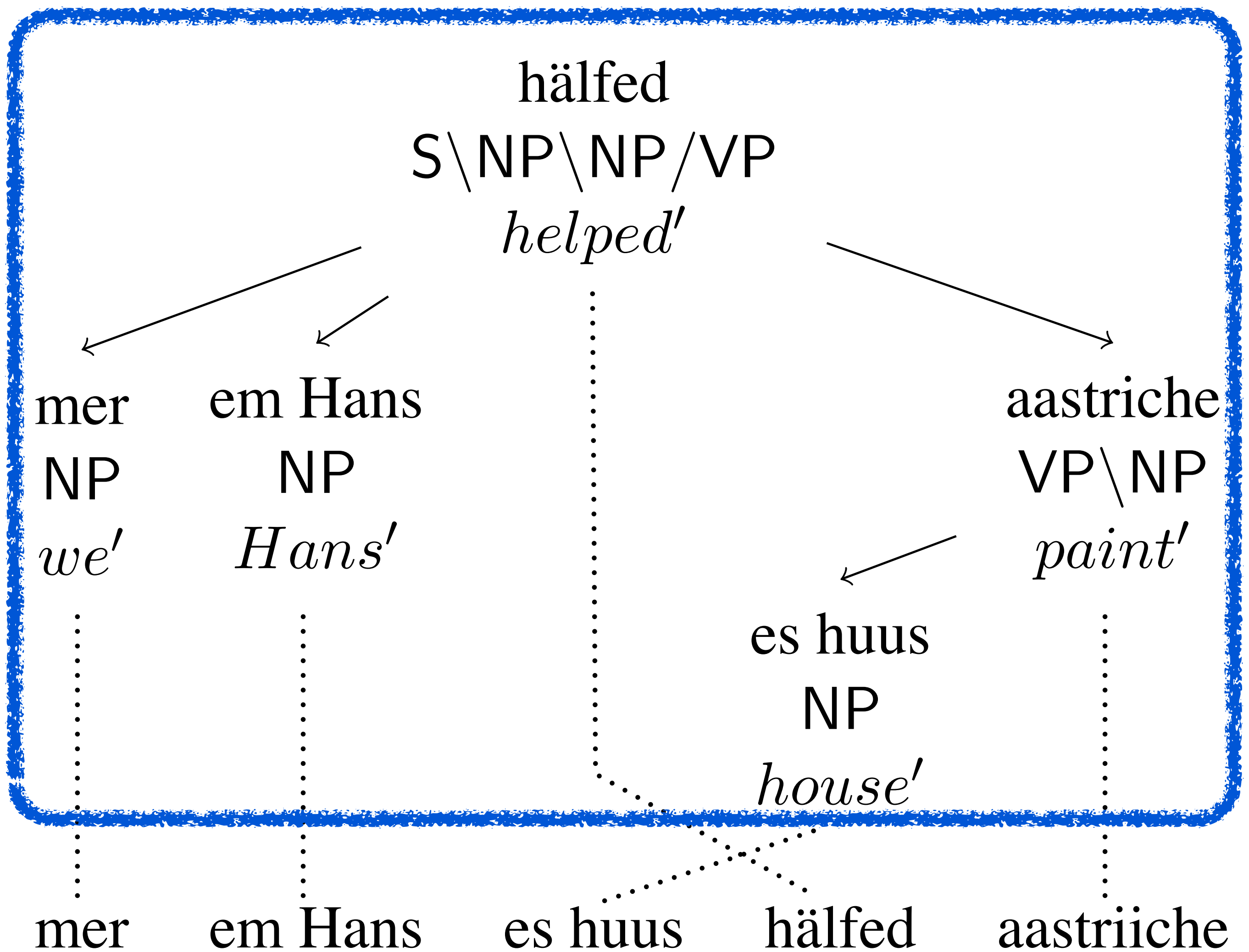
Set-theoretic view

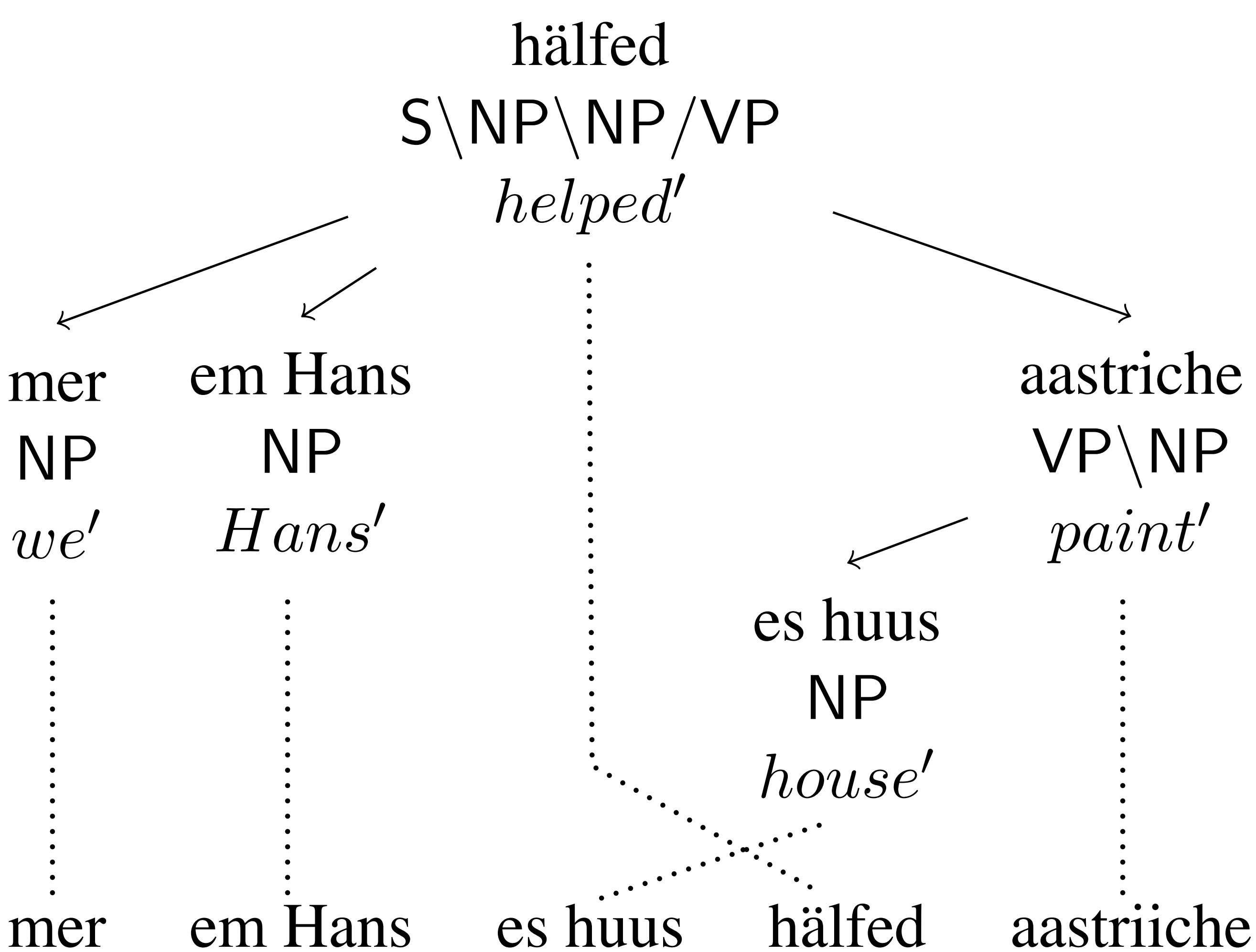


Set-theoretic view



CCG valency tree (Koller & Kuhlmann 2009)

Set-theoretic view



Set-theoretic view

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$$\begin{bmatrix} \text{es huus} \vdash \mathsf{NP} : house' \\ \text{the house} \vdash \mathsf{NP} : house' \end{bmatrix}$$

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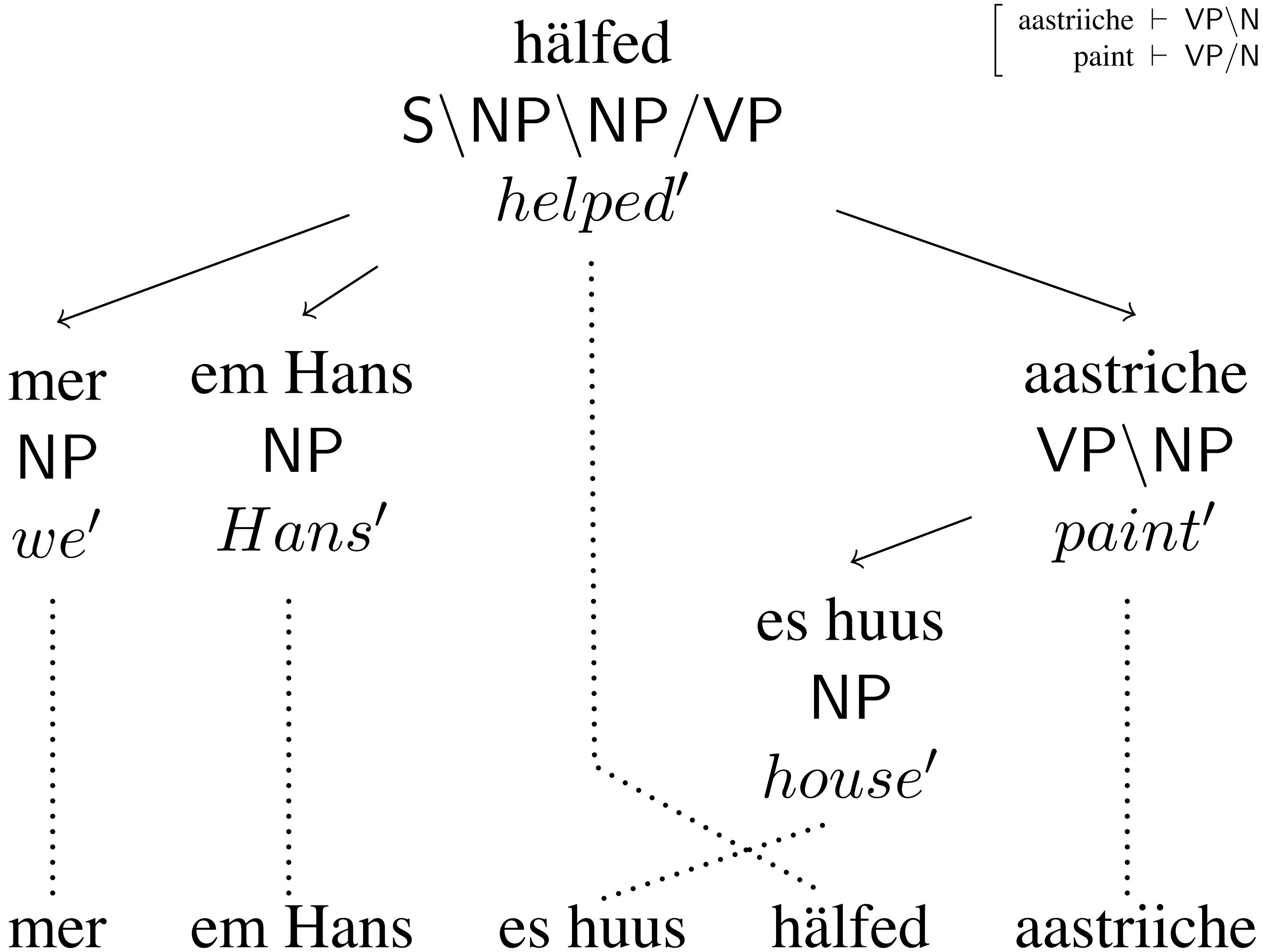
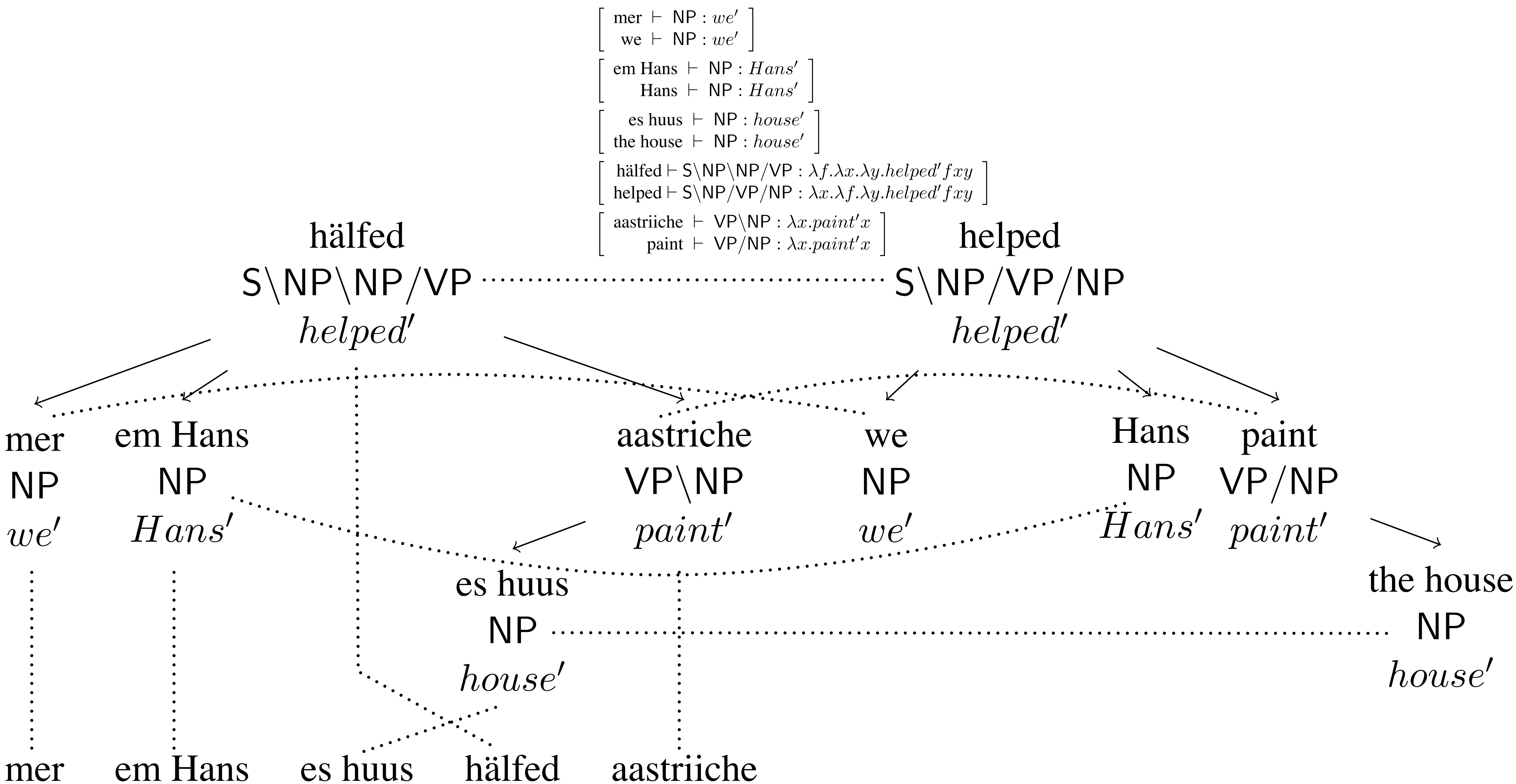
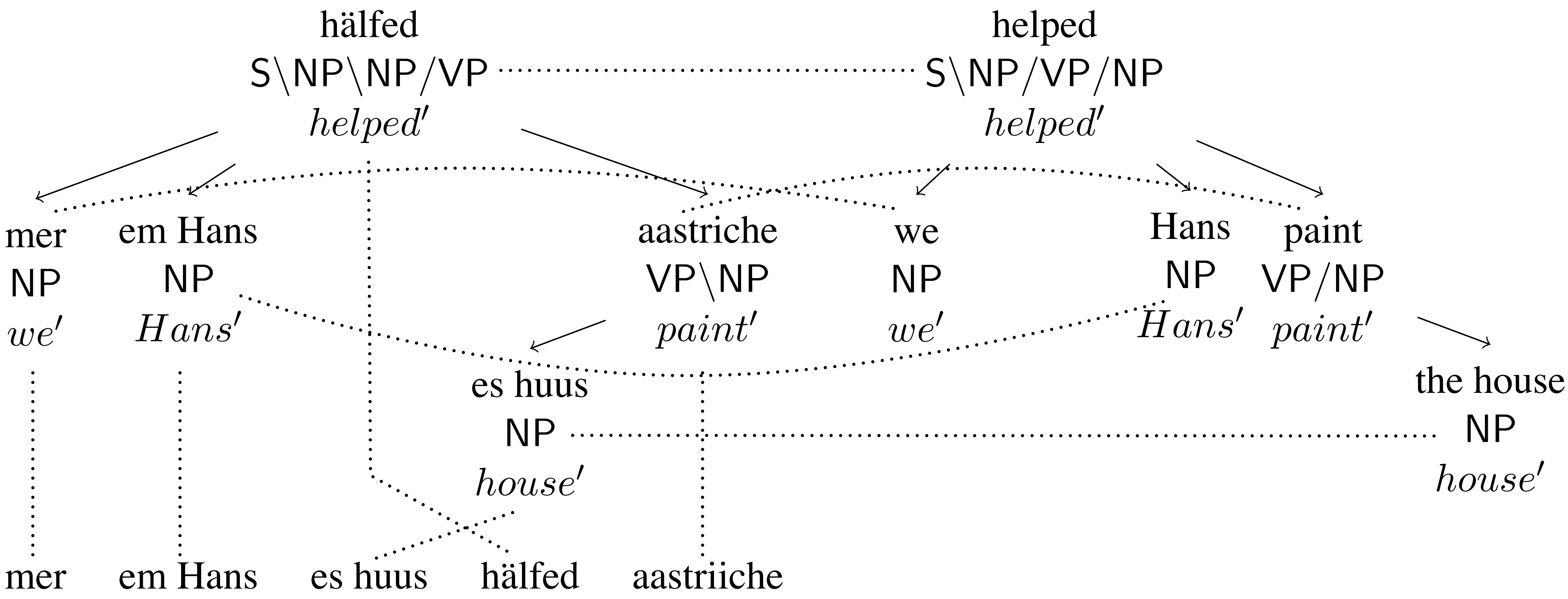


Set-theoretic view



Set-theoretic view



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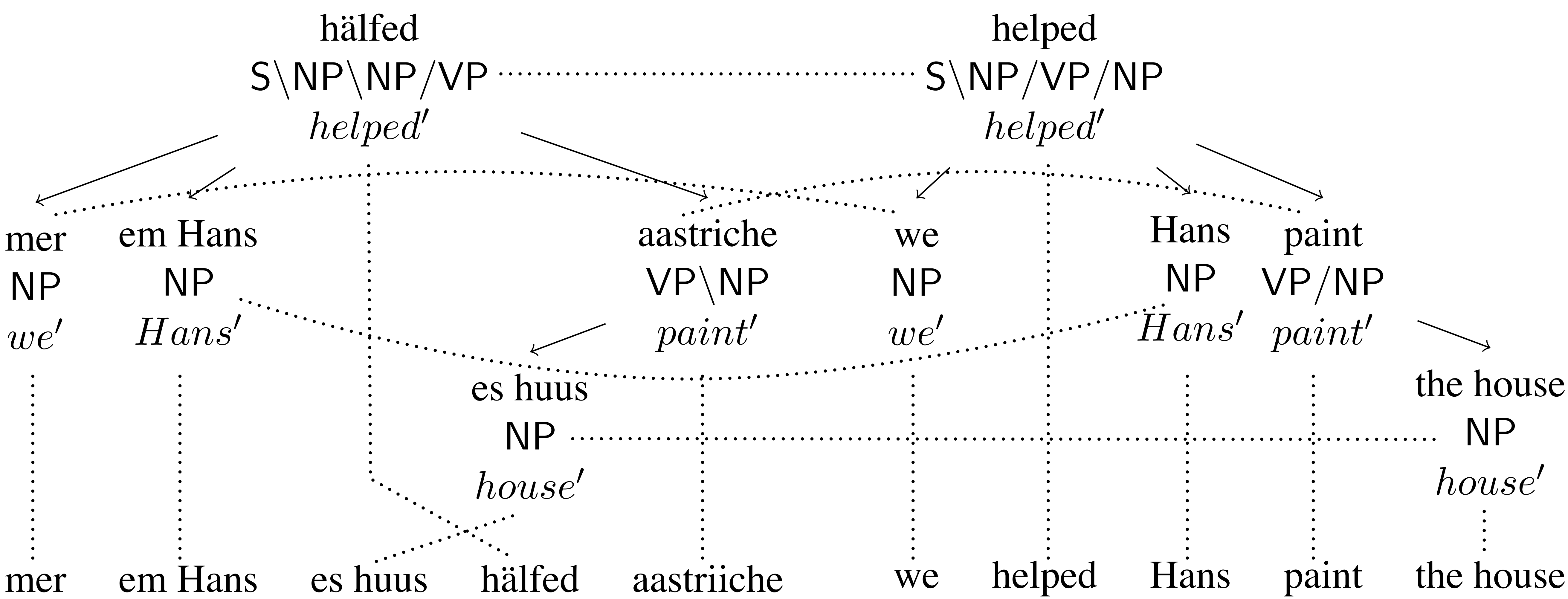

SCCG recognition by intersection

- Given SCGG $\mathcal{G}$ and string pair u, v:
  - Construct a CCG $\mathcal{G}'_L$ producing all and only the set of valency trees of derivations of u.
  - Project the lexical categories of $\mathcal{G}'_L$ through the synchronous lexicon to obtain CCG $\mathcal{G}'_R$.
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Bar-Hillel construction for CCG?

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Mark Steedman steedman@inf.ed.ac.uk via cs.jhu.e 12/13/12

to alopez

Hi, Adam

> is there a constructive proof that CCG is closed under intersection
> with regular languages?

I don't know of one. Its easy to show (as informally in my 2000 book) that for every CCG there is a weakly equivalent LIG. However, it's not so easy to show the reverse, and it isn't entirely clear whether that is actually the case. So I guess it doesn't follow from

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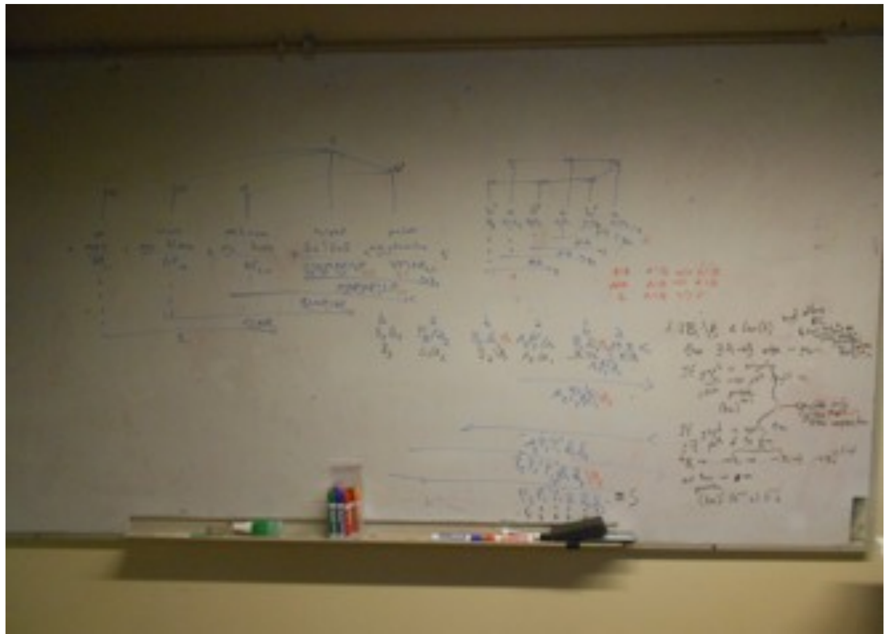

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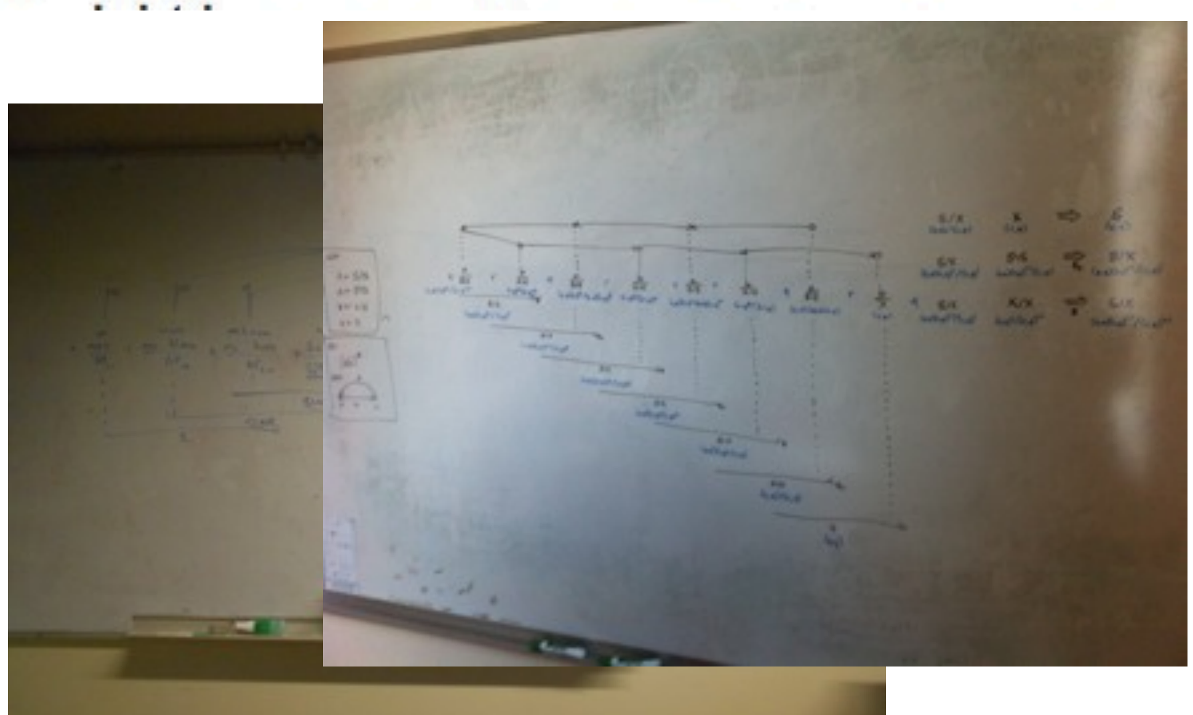

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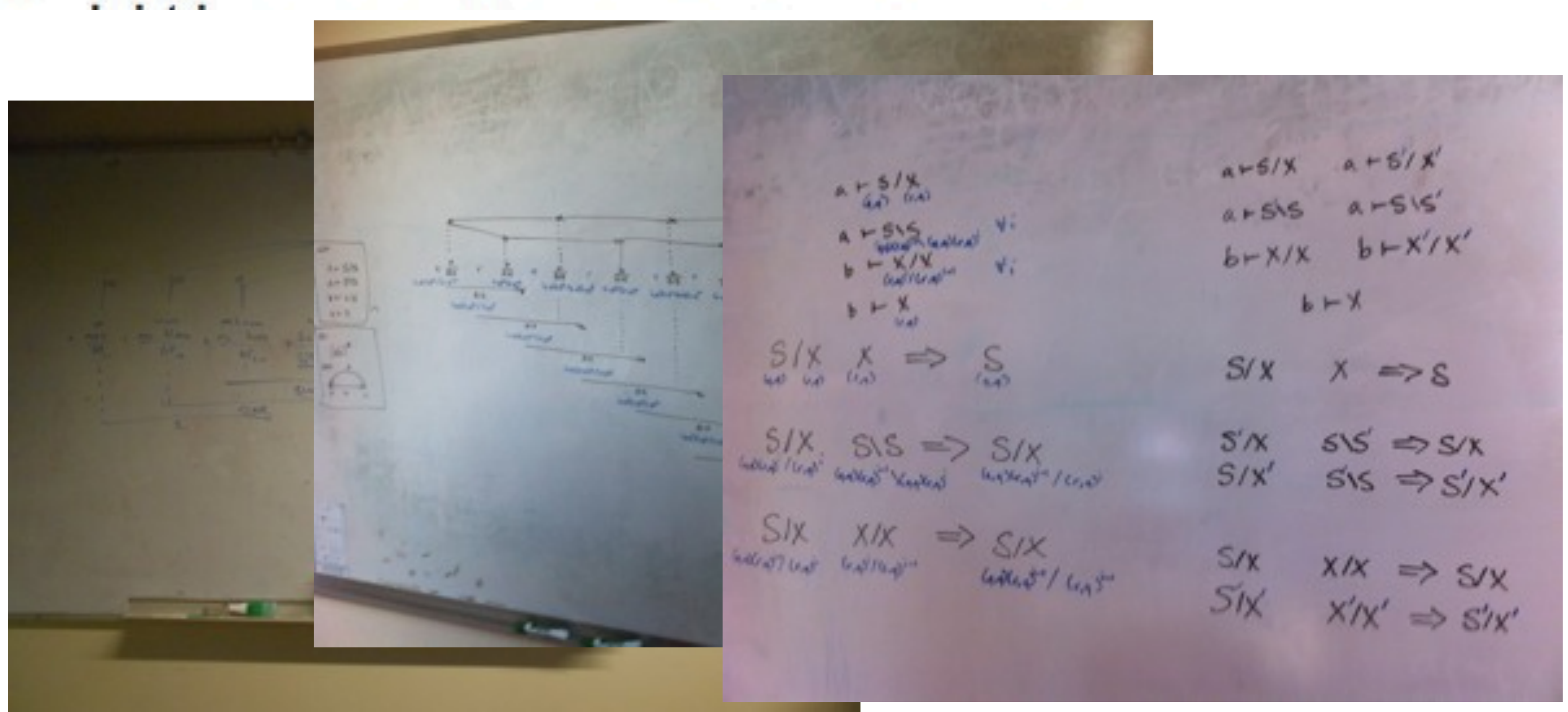

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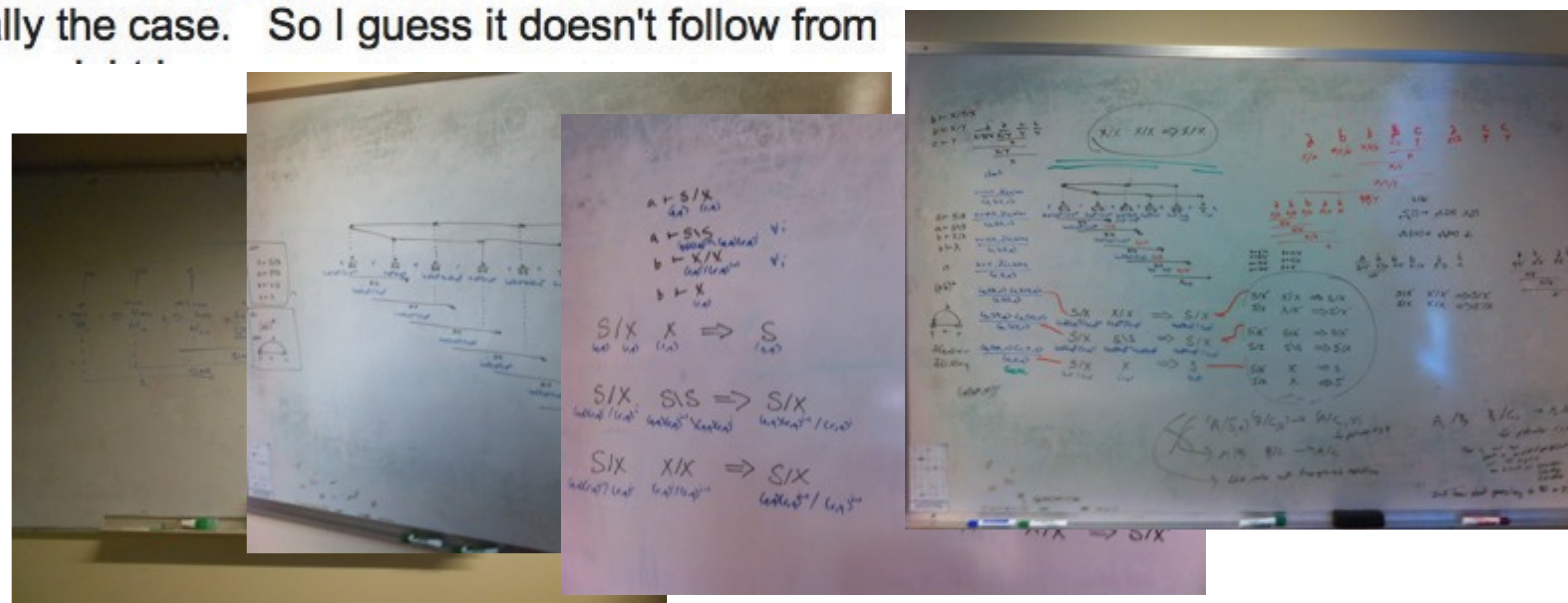
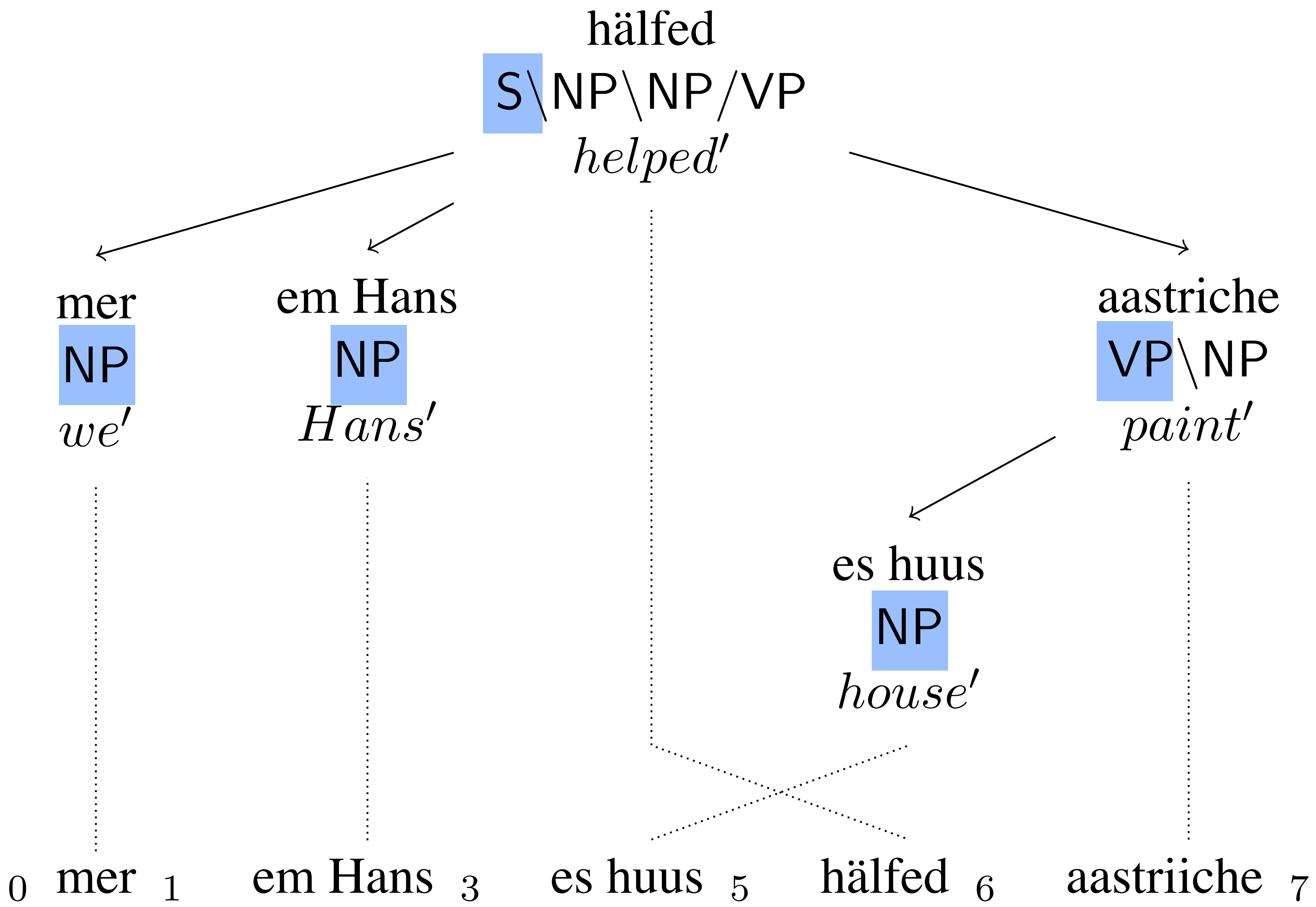


Easier: CCG intersected with *finite* language

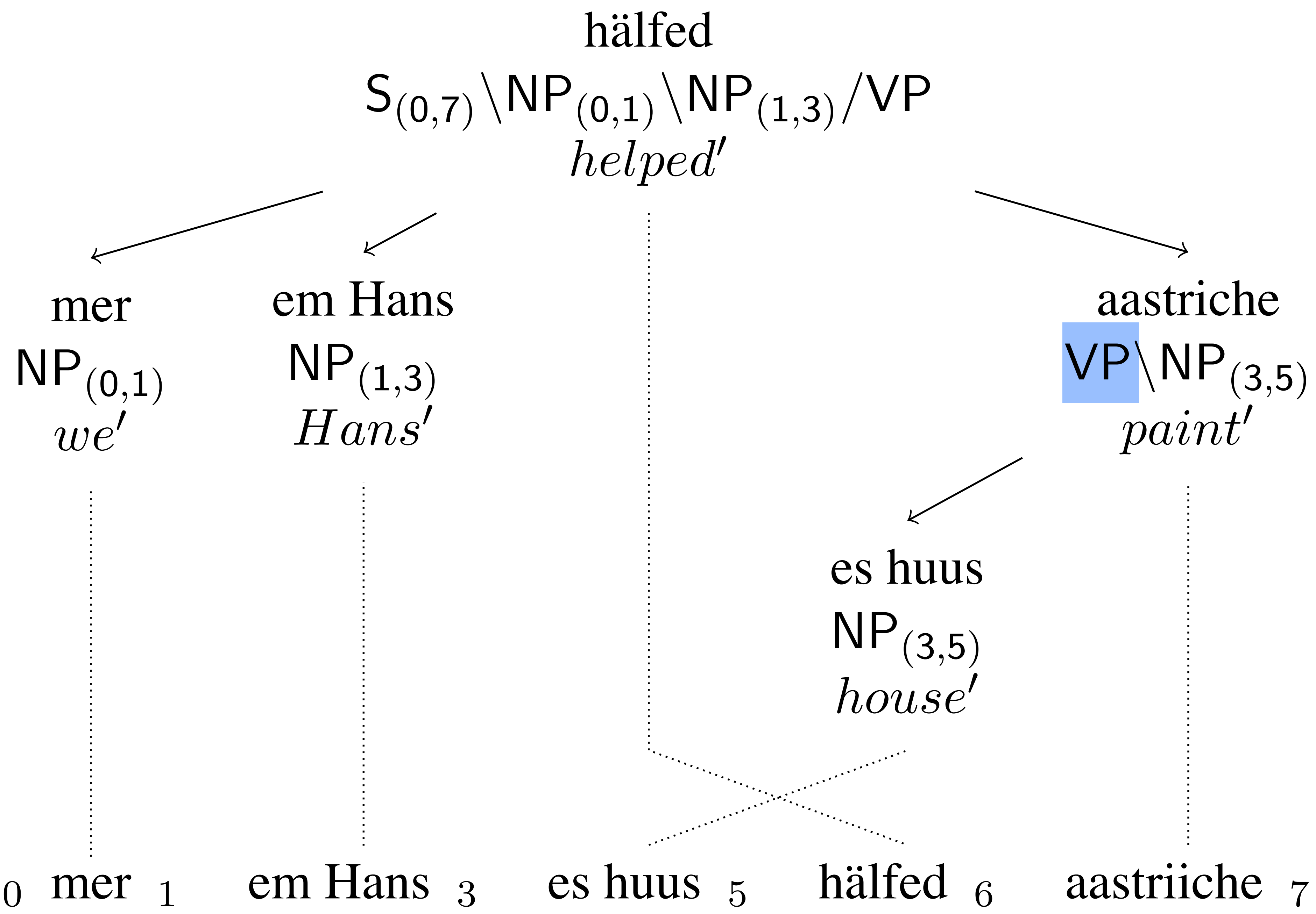
- Represent finite language with an *acyclic* NFA.
- Can represent exponentially many strings, as in speech recognizer output or segmentation/tokenization lattices.

Intuitions



hälfed
S\NP\NP/VP
helped′
mer
NP
we′
em Hans
NP
Hans′
aastriche
VP\NP
paint′
es huus
NP
house′
0 mer 1 em Hans 3 es huus 5 hälfed 6 aastriiche 7

Intuitions



hälfed
$S_{(0,7)}\backslash NP_{(0,1)}\backslash NP_{(1,3)}/VP$
$helped'$
mer
$NP_{(0,1)}$
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em Hans
$NP_{(1,3)}$
$Hans'$
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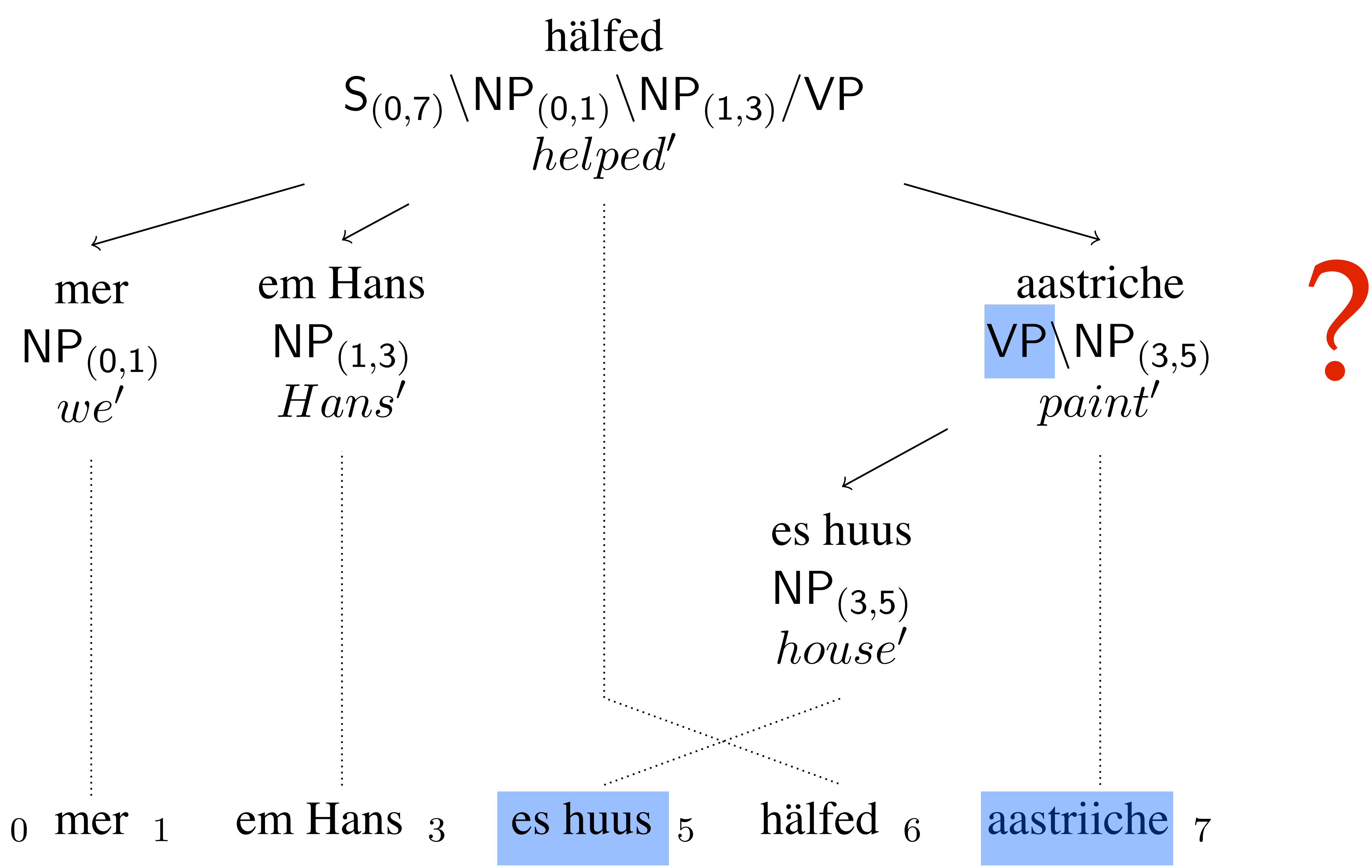

hälfed
S(0,7)\NP(0,1)\NP(1,3)/VP
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NP(3,5)
house′
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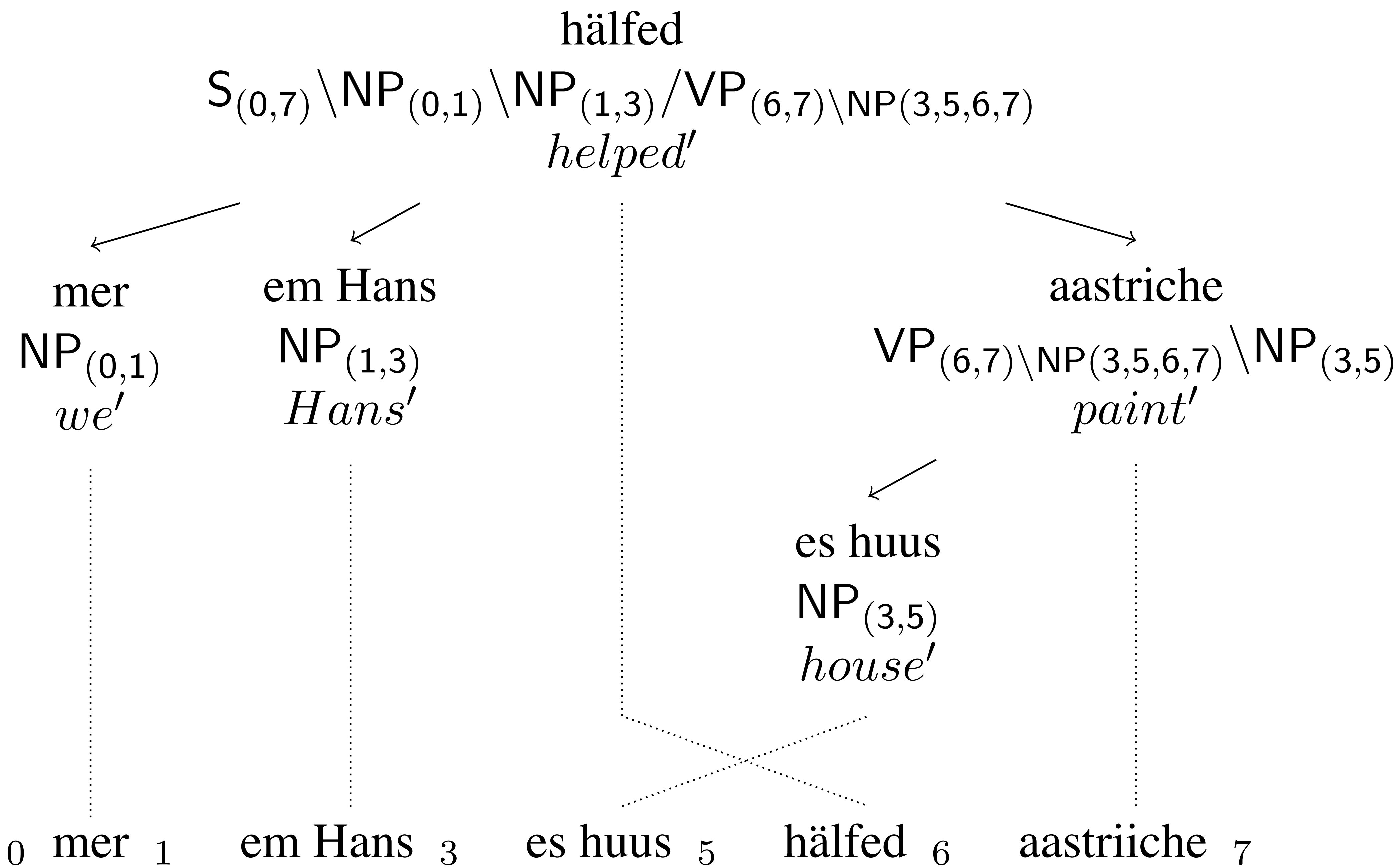
Intuitions


hälfed
$S_{(0,7)}\backslash NP_{(0,1)}\backslash NP_{(1,3)}/VP_{(6,7)\backslash NP(3,5,6,7)}$
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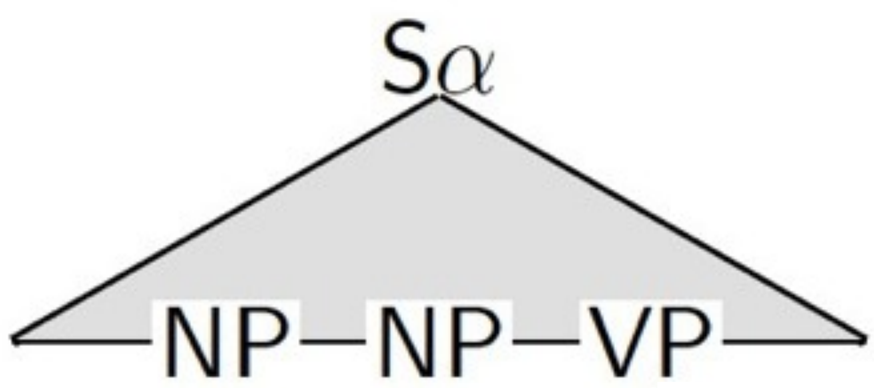
$VP_{(6,7)\backslash NP(3,5,6,7)}\backslash NP_{(3,5)}$
$paint'$
es huus
$NP_{(3,5)}$
$house'$
$_0$ mer $_1$ em Hans $_3$ es huus $_5$ hälfed $_6$ aastriiche $_7$

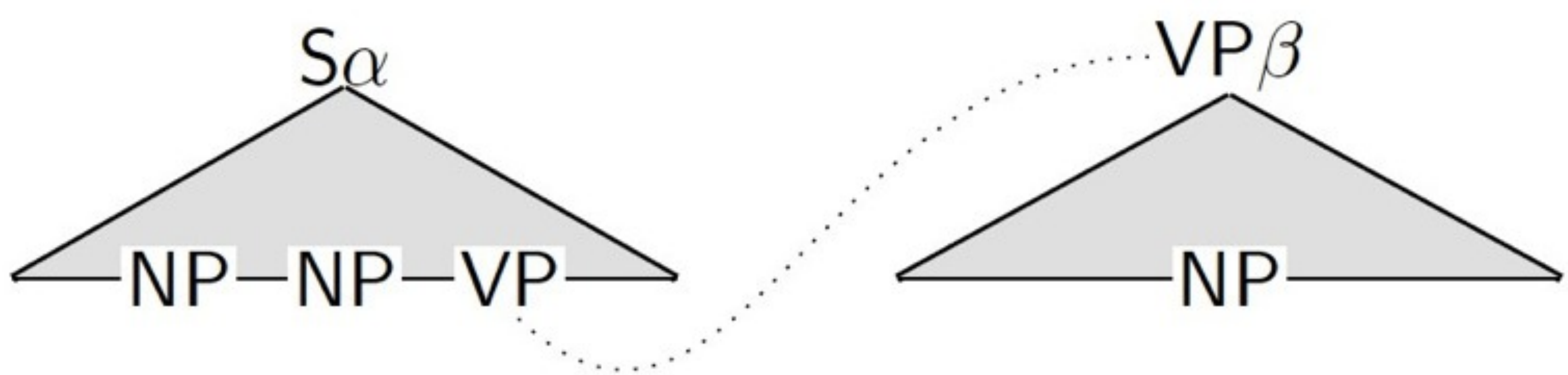
Compositional behavior of categories

$S\backslash NP\backslash NP/VP$



Compositional behavior of categories

$$\mathsf{S\backslash NP\backslash NP/VP} \quad \mathsf{VP\backslash NP} \Rightarrow \mathsf{S\backslash NP\backslash NP\backslash NP}$$



Compositional behavior of categories

mer | em Hans | es huus | hälfed | aastriiche

$$\mathsf{NP} : we' \qquad \mathsf{NP} : Hans' \qquad \mathsf{NP} : house' \qquad \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'fxy \qquad \mathsf{VP\backslash NP} : \lambda x.paint'x$$

$$\xrightarrow{>\mathbf{B}_\times} \quad \mathsf{S\backslash NP\backslash NP\backslash NP} : \lambda z.\lambda x.\lambda y.helped'(paint'z)xy$$

$$\xrightarrow{<} \quad \mathsf{S\backslash NP\backslash NP} : \lambda x.\lambda y.helped'(paint'house')xy$$

$$\xrightarrow{<} \quad \mathsf{S\backslash NP} : \lambda y.helped'(paint'house')Hans'y$$

$$\xrightarrow{<} \quad \mathsf{S} : helped'(paint'house')Hans'we'$$

Compositional behavior of categories

mer	em Hans	es huus	hälfed	aastriiche
$\mathsf{NP} : we'$	$\mathsf{NP} : Hans'$	$\mathsf{NP} : house'$	$\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'fxy$	$\mathsf{VP\backslash NP} : \lambda x.paint'x$

$$\mathsf{S\backslash NP\backslash NP\backslash NP} : \lambda z.\lambda x.\lambda y.helped'(paint'z)xy \quad (>\mathbf{B}_\times)$$

$$\mathsf{S\backslash NP\backslash NP} : \lambda x.\lambda y.helped'(paint'house')xy \quad (<)$$

$$\mathsf{S\backslash NP} : \lambda y.helped'(paint'house')Hans'y \quad (<)$$

$$\mathsf{S} : helped'(paint'house')Hans'we' \quad (<)$$

$$\mathsf{VP}_{(6,7)}$$

Compositional behavior of categories

mer	em Hans	es huus	hälfed	aastriiche
$\mathsf{NP} : we'$	$\mathsf{NP} : Hans'$	$\mathsf{NP} : house'$	$\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'\,fxy$	$\mathsf{VP\backslash NP} : \lambda x.paint'\,x$

$$\mathsf{S\backslash NP\backslash NP\backslash NP} : \lambda z.\lambda x.\lambda y.helped'(paint'\,z)xy \quad >\mathbf{B}_\times$$

$$\mathsf{S\backslash NP\backslash NP} : \lambda x.\lambda y.helped'(paint'\,house')xy \quad <$$

$$\mathsf{S\backslash NP} : \lambda y.helped'(paint'\,house')Hans'\,y \quad <$$

$$\mathsf{S} : helped'(paint'\,house')Hans'\,we' \quad <$$

$$\mathsf{VP}_{(6,7)\backslash \mathsf{NP}}$$

Compositional behavior of categories

mer	em Hans	es huus	hälfed	aastriiche
$\mathsf{NP} : we'$	$\mathsf{NP} : Hans'$	$\mathsf{NP} : house'$	$\mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'\,fxy$	$\mathsf{VP\backslash NP} : \lambda x.paint'\,x$

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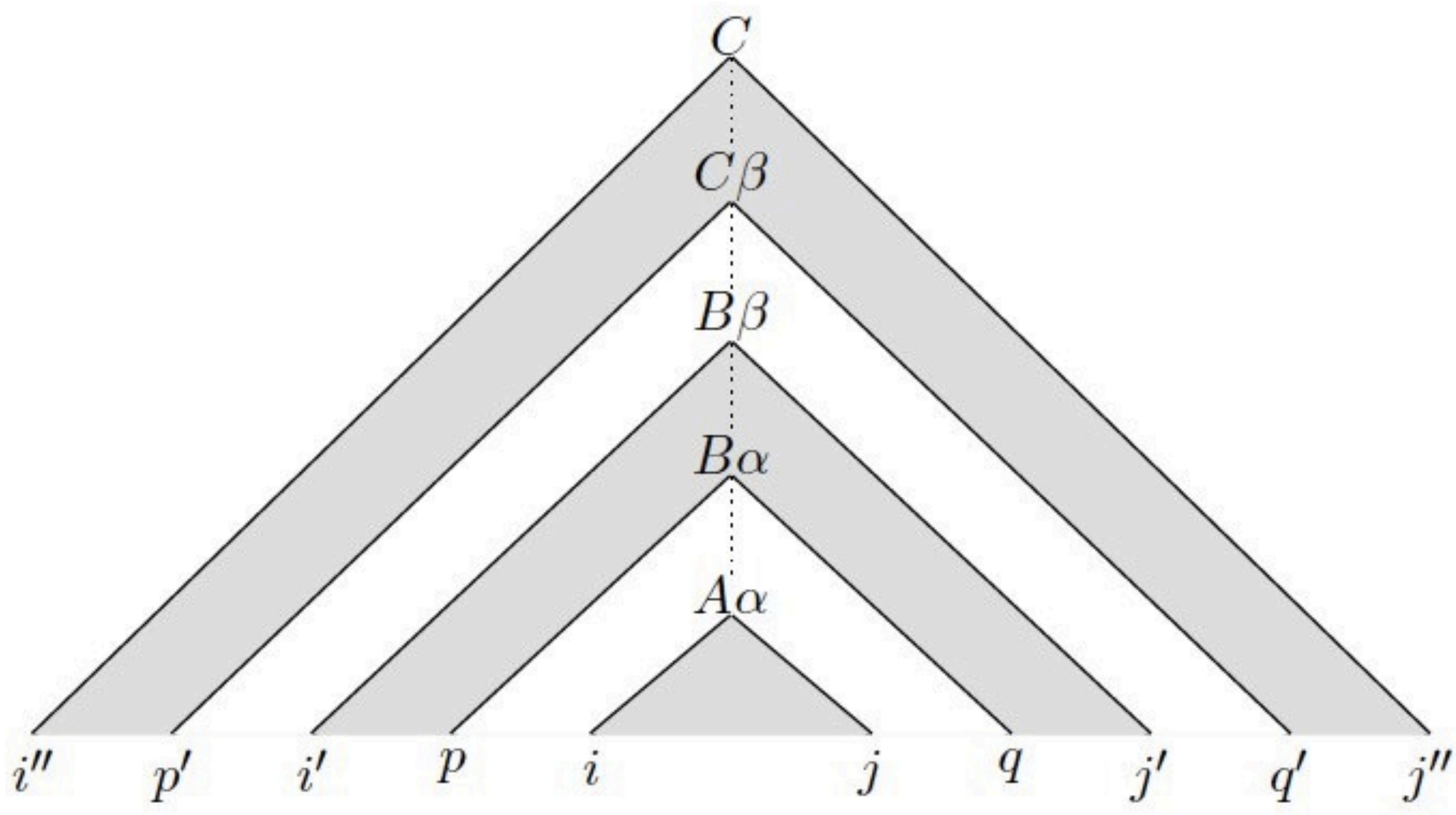
$$\mathsf{S\backslash NP} : \lambda y.helped'(paint'\,house')Hans'y \quad <$$

$$\mathsf{S} : helped'(paint'\,house')Hans'we' \quad <$$

$$\mathsf{VP}_{(6,7)}\backslash \mathsf{NP}(3,5,6,7)$$

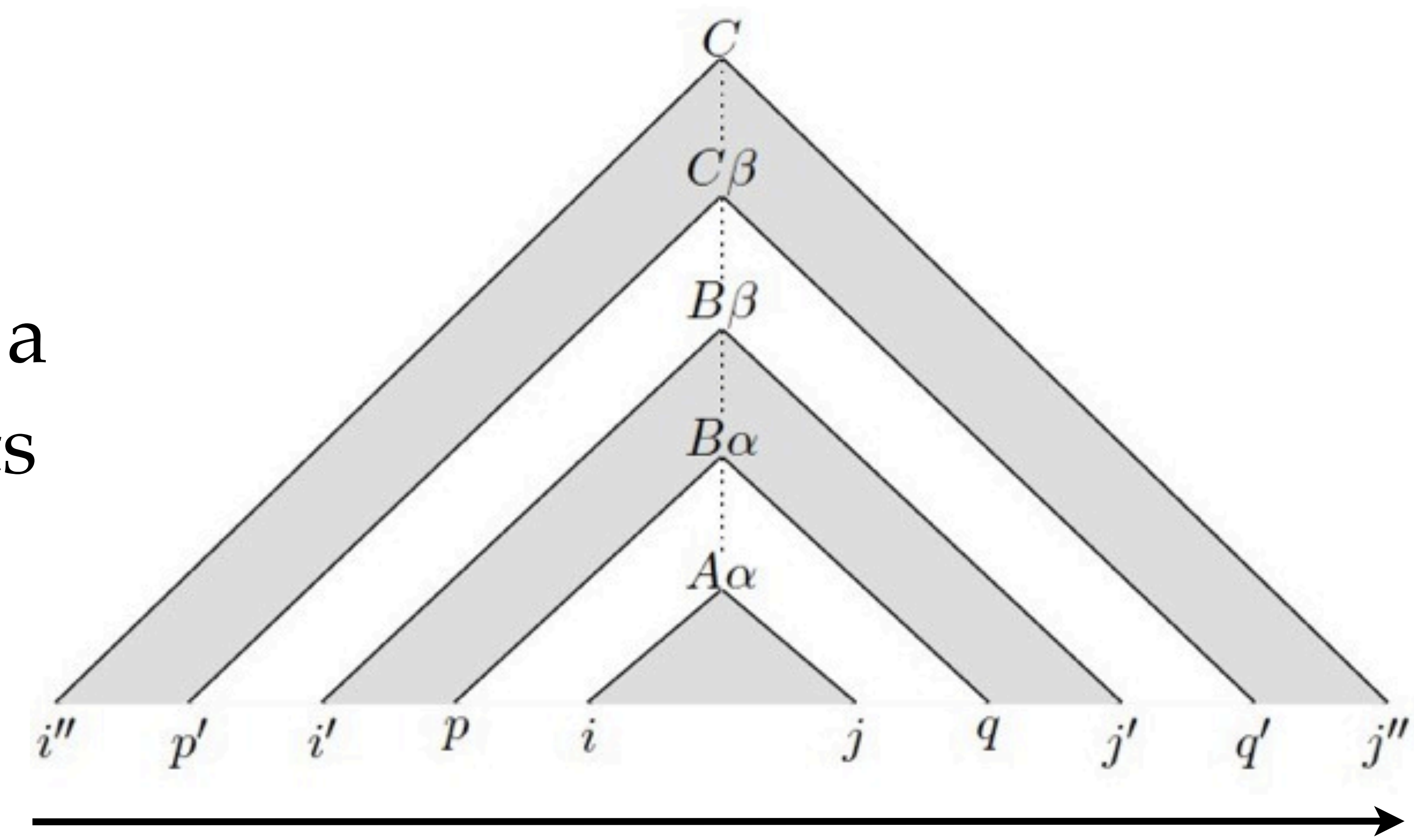
Yield of a CCG category



Summary: $(i, j)\alpha(i', (p, q), j')\beta(i'', (p', q'), j'')$

Construction

For all sequences of states s.t. that there is a path in DFA that visits in this order:



and all sequences of up to *n* arguments $\alpha, \beta, \gamma\ldots.$

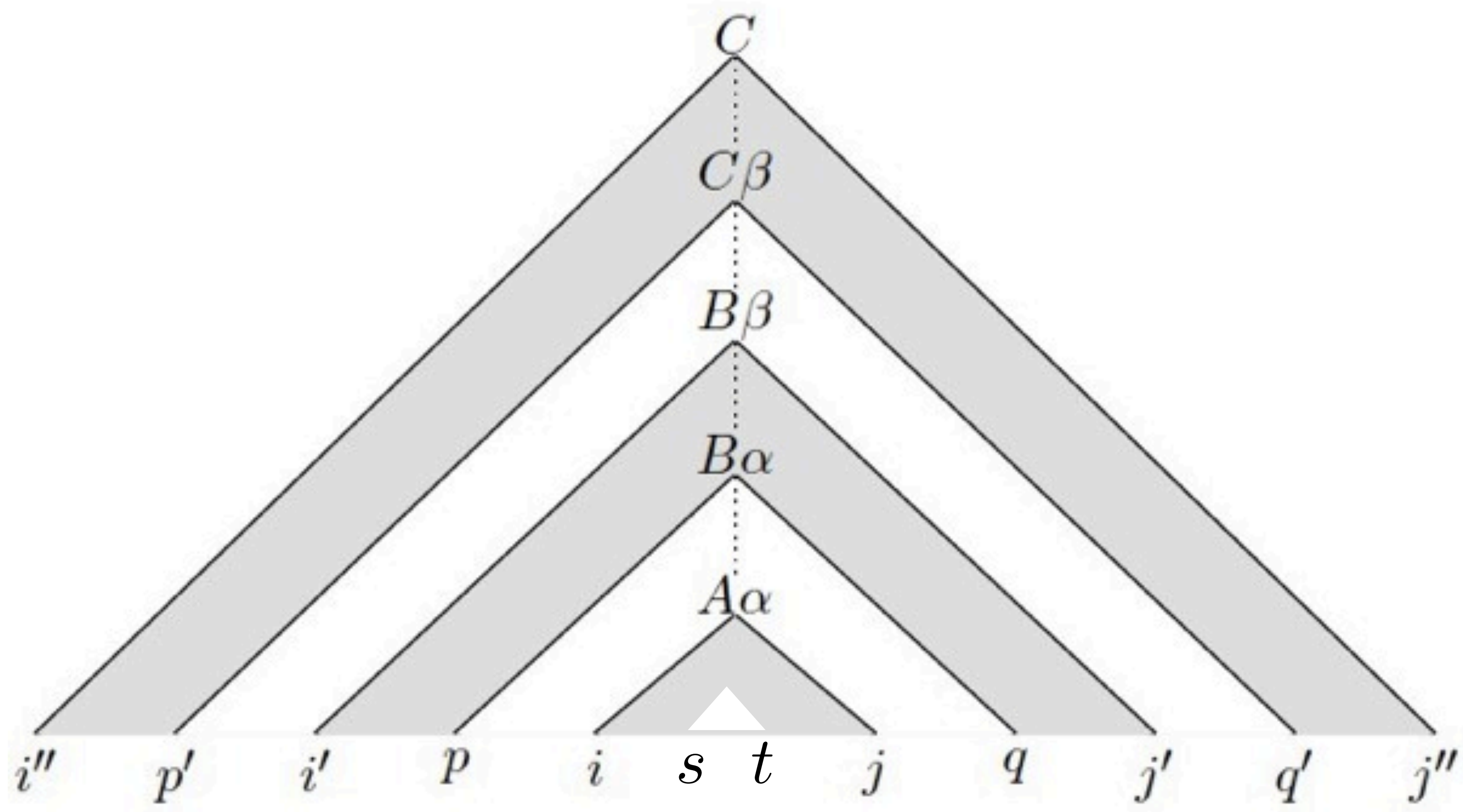
and all categories *A*

$A_{(i,j)\alpha(i',(p,q),j')\beta(i'',(p',q'),j'')}$ is a category in $\mathcal{G}_L$

Construction

If $w \vdash A/B$ is in $\mathcal{G}$

and $t \in \delta(s, w)$



Then

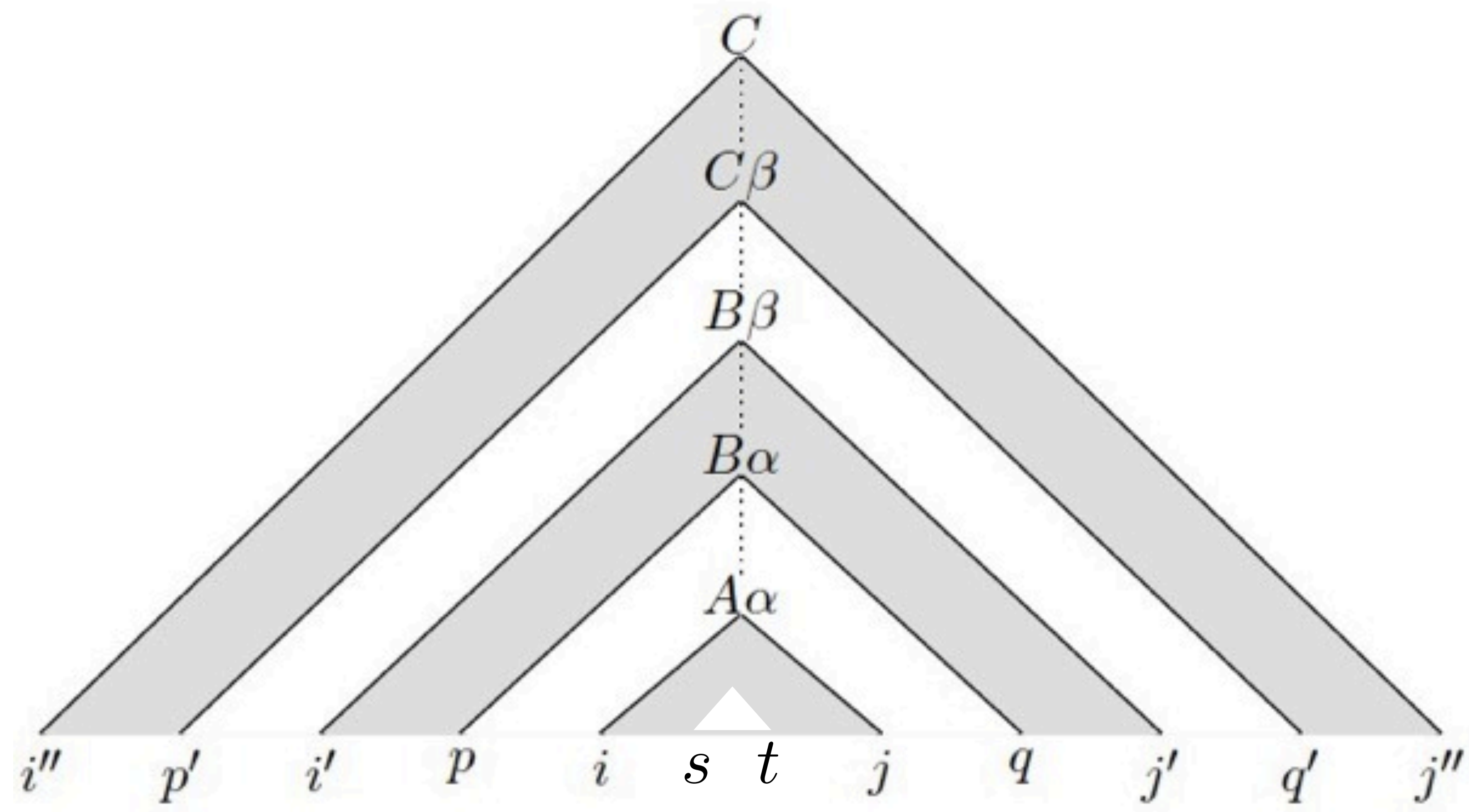
$$w \vdash A_{(i,j)\alpha(i',(p,q),j')\beta(i'',(p',q'),j'')} / B_{(t,j)\gamma(i,s)\alpha(i',(p,q,),j')\beta(i'',(p',q'),j'')}$$

is in $\mathcal{G}_L$

Construction

If $w \vdash A/B$ is in $\mathcal{G}$

and $t \in \delta(s, w)$



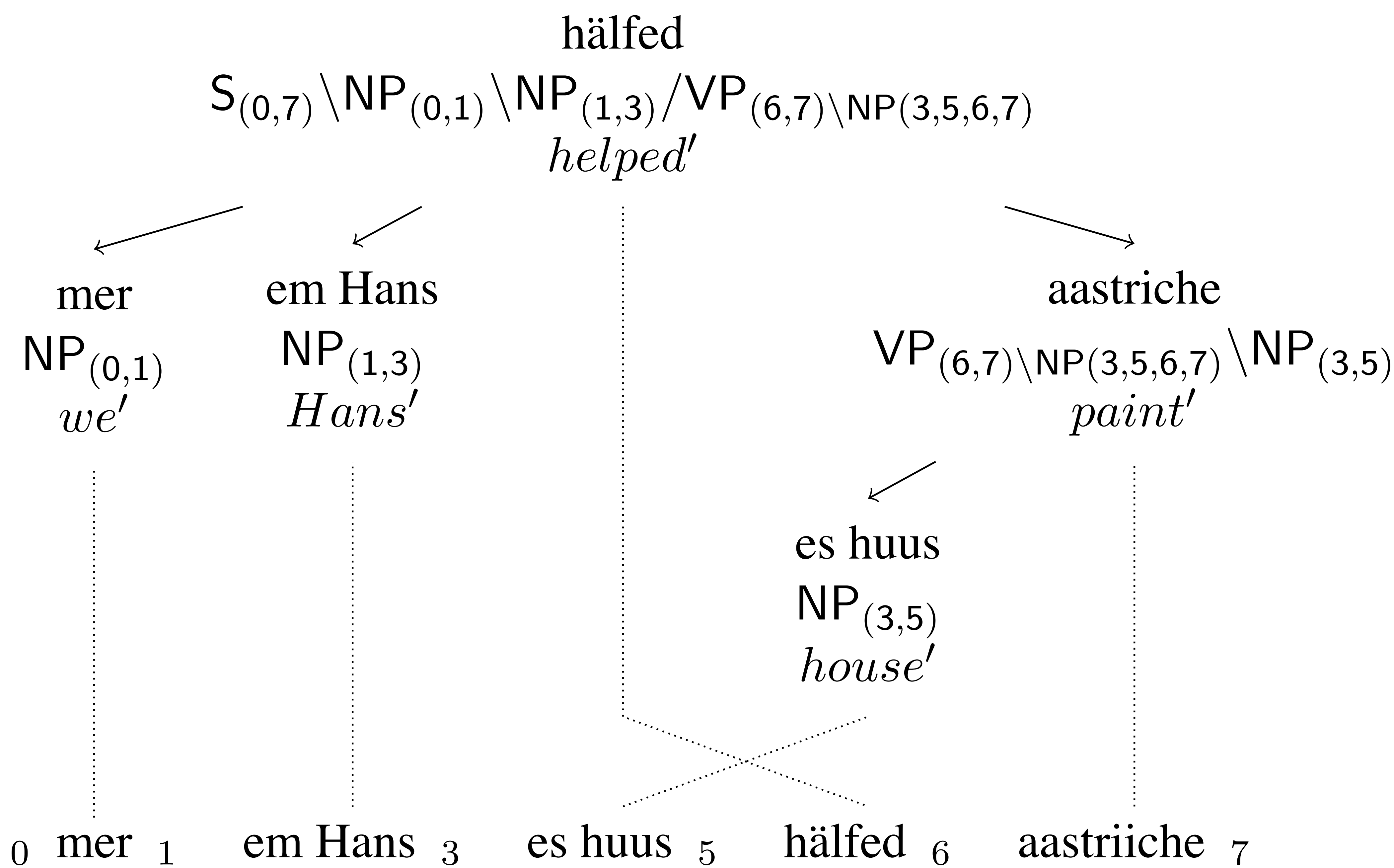
Then

$$w \vdash A_{(i,j)\alpha(i',(p,q),j')\beta(i'',(p',q'),j'')}/$$

$$B_{(t,j)\gamma(i,s)\alpha(i',(p,q,),j')\beta(i'',(p',q'),j'')}$$

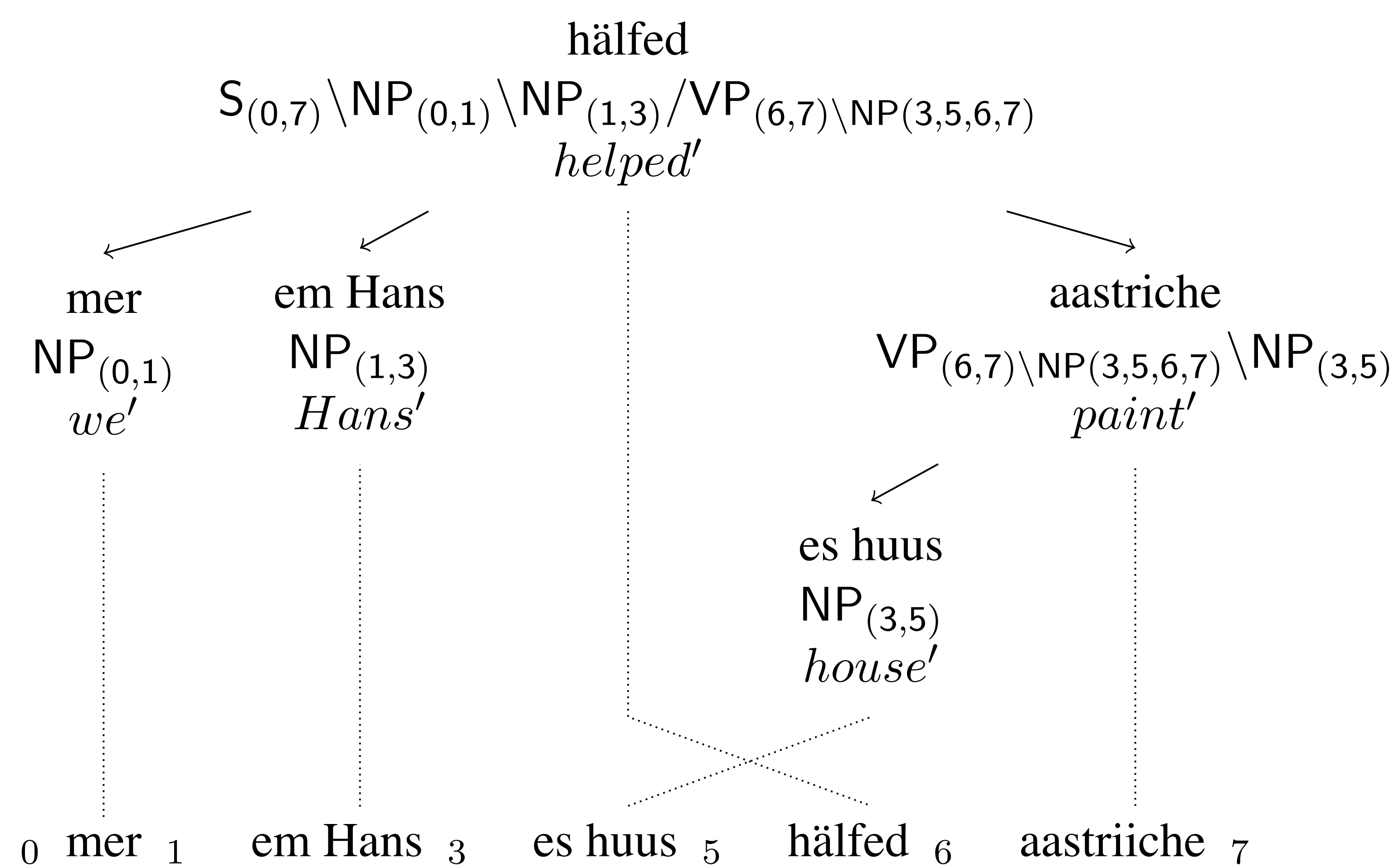
is in $\mathcal{G}_L$ ...and spans (s, t)

Recognition Again



Recognition Again

$\mathcal{G}'$ produces only strings with this valency tree



$$
\begin{aligned}
\text{mer} &\vdash NP_{(0,1)} : we' \\
\text{em Hans} &\vdash NP_{(1,3)} : Hans' \\
\text{es huus} &\vdash NP_{(3,5)} : house' \\
\text{hälfed} &\vdash S_{(0,7)}\backslash NP_{(0,1)}\backslash NP_{(1,3)}/VP_{(6,7)\backslash NP(3,5,6,7)} : \lambda f.\lambda x.\lambda y.helped' fxy \\
\text{aastriiche} &\vdash VP_{(6,7)\backslash NP,(3,5)}\backslash NP_{(3,5)} : \lambda x.paint' x
\end{aligned}
$$

Recognition Again

$$\left[\begin{array}{r} \text{mer} \vdash \mathsf{NP} : we' \\ \text{we} \vdash \mathsf{NP} : we' \end{array}\right]$$

$$\left[\begin{array}{r} \text{em Hans} \vdash \mathsf{NP} : Hans' \\ \text{Hans} \vdash \mathsf{NP} : Hans' \end{array}\right]$$

$$\left[\begin{array}{r} \text{es huus} \vdash \mathsf{NP} : house' \\ \text{the house} \vdash \mathsf{NP} : house' \end{array}\right]$$

$$\left[\begin{array}{r} \text{hälfed} \vdash \mathsf{S \backslash NP \backslash NP / VP} : \lambda f.\lambda x.\lambda y.helped' f x y \\ \text{helped} \vdash \mathsf{S \backslash NP / VP / NP} : \lambda x.\lambda f.\lambda y.helped' f x y \end{array}\right]$$

$$\left[\begin{array}{r} \text{aastriiche} \vdash \mathsf{VP \backslash NP} : \lambda x.paint' x \\ \text{paint} \vdash \mathsf{VP / NP} : \lambda x.paint' x \end{array}\right]$$

$$\begin{array}{r l} \text{mer} & \vdash \mathsf{NP}_{(0,1)} : we' \\ \text{em Hans} & \vdash \mathsf{NP}_{(1,3)} : Hans' \\ \text{es huus} & \vdash \mathsf{NP}_{(3,5)} : house' \\ \text{hälfed} & \vdash \mathsf{S}_{(0,7)} \backslash \mathsf{NP}_{(0,1)} \backslash \mathsf{NP}_{(1,3)} / \mathsf{VP}_{(6,7) \backslash \mathsf{NP}(3,5,6,7)} : \lambda f.\lambda x.\lambda y.helped' f x y \\ \text{aastriiche} & \vdash \mathsf{VP}_{(6,7) \backslash \mathsf{NP},(3,5)} \backslash \mathsf{NP}_{(3,5)} : \lambda x.paint' x \end{array}$$

Recognition Again

$$\begin{bmatrix} \text{mer} \vdash \mathsf{NP} : we' \\ \text{we} \vdash \mathsf{NP} : we' \end{bmatrix}$$

$$\begin{bmatrix} \text{em Hans} \vdash \mathsf{NP} : Hans' \\ \text{Hans} \vdash \mathsf{NP} : Hans' \end{bmatrix}$$

$$\begin{bmatrix} \text{es huus} \vdash \mathsf{NP} : house' \\ \text{the house} \vdash \mathsf{NP} : house' \end{bmatrix}$$

$$\begin{bmatrix} \text{hälfed} \vdash \mathsf{S}\backslash\mathsf{NP}\backslash\mathsf{NP}/\mathsf{VP} : \lambda f.\lambda x.\lambda y.helped'\, fxy \\ \text{helped} \vdash \mathsf{S}\backslash\mathsf{NP}/\mathsf{VP}/\mathsf{NP} : \lambda x.\lambda f.\lambda y.helped'\, fxy \end{bmatrix}$$

$$\begin{bmatrix} \text{aastriiche} \vdash \mathsf{VP}\backslash\mathsf{NP} : \lambda x.paint'x \\ \text{paint} \vdash \mathsf{VP}/\mathsf{NP} : \lambda x.paint'x \end{bmatrix}$$

$$\text{mer} \vdash \mathsf{NP}_{(0,1)} : we'$$

$$\text{em Hans} \vdash \mathsf{NP}_{(1,3)} : Hans'$$

$$\text{es huus} \vdash \mathsf{NP}_{(3,5)} : house'$$

$$\text{hälfed} \vdash \mathsf{S}_{(0,7)}\backslash\mathsf{NP}_{(0,1)}\backslash\mathsf{NP}_{(1,3)}/\mathsf{VP}_{(6,7)\backslash\mathsf{NP}(3,5,6,7)} : \lambda f.\lambda x.\lambda y.helped'\, fxy$$

$$\text{aastriiche} \vdash \mathsf{VP}_{(6,7)\backslash\mathsf{NP},(3,5)}\backslash\mathsf{NP}_{(3,5)} : \lambda x.paint'x$$

Recognition Again

$$\left[\begin{array}{rl} \text{mer} \vdash & \mathsf{NP} : we' \\ \text{we} \vdash & \mathsf{NP} : we' \end{array}\right]$$

$$\left[\begin{array}{rl} \text{em Hans} \vdash & \mathsf{NP} : Hans' \\ \text{Hans} \vdash & \mathsf{NP} : Hans' \end{array}\right]$$

$$\left[\begin{array}{rl} \text{es huus} \vdash & \mathsf{NP} : house' \\ \text{the house} \vdash & \mathsf{NP} : house' \end{array}\right]$$

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$$\left[\begin{array}{rl} \text{aastriiche} \vdash & \mathsf{VP}\backslash\mathsf{NP} : \lambda x.paint' x \\ \text{paint} \vdash & \mathsf{VP}/\mathsf{NP} : \lambda x.paint' x \end{array}\right]$$

$$\begin{array}{r} \text{we} \vdash \mathsf{NP}_{(0,1)} : we' \\ \text{Hans} \vdash \mathsf{NP}_{(1,3)} : Hans' \\ \text{the house} \vdash \mathsf{NP}_{(3,5)} : house' \\ \text{helped} \vdash \mathsf{S}_{(0,7)}\backslash\mathsf{NP}_{(0,1)}/\mathsf{VP}_{(6,7)\backslash\mathsf{NP}(3,5,6,7)}/\mathsf{NP}_{(1,3)} : \lambda x.\lambda f.\lambda y.helped' fxy \\ \text{paint} \vdash \mathsf{VP}_{(6,7)\backslash\mathsf{NP},(3,5)}/\mathsf{NP}_{(3,5)} : \lambda x.paint' x \end{array}$$

Recognition Again

$$\begin{bmatrix} \text{mer} \vdash \mathsf{NP} : we' \\ \text{we} \vdash \mathsf{NP} : we' \end{bmatrix}$$

$$\begin{bmatrix} \text{em Hans} \vdash \mathsf{NP} : Hans' \\ \text{Hans} \vdash \mathsf{NP} : Hans' \end{bmatrix}$$

$$\begin{bmatrix} \text{es huus} \vdash \mathsf{NP} : house' \\ \text{the house} \vdash \mathsf{NP} : house' \end{bmatrix}$$

$$\begin{bmatrix} \text{hälfed} \vdash \mathsf{S\backslash NP\backslash NP/VP} : \lambda f.\lambda x.\lambda y.helped'\,fxy \\ \text{helped} \vdash \mathsf{S\backslash NP/VP/NP} : \lambda x.\lambda f.\lambda y.helped'\,fxy \end{bmatrix}$$

$$\begin{bmatrix} \text{aastriiche} \vdash \mathsf{VP\backslash NP} : \lambda x.paint'x \\ \text{paint} \vdash \mathsf{VP/NP} : \lambda x.paint'x \end{bmatrix}$$

$$\text{we} \vdash \mathsf{NP}_{(0,1)} : we'$$

$$\text{Hans} \vdash \mathsf{NP}_{(1,3)} : Hans'$$

$$\text{the house} \vdash \mathsf{NP}_{(3,5)} : house'$$

$$\text{helped} \vdash \mathsf{S}_{(0,7)}\backslash\mathsf{NP}_{(0,1)}/\mathsf{VP}_{(6,7)\backslash \mathsf{NP}(3,5,6,7)}/\mathsf{NP}_{(1,3)} : \lambda x.\lambda f.\lambda y.helped'\,fxy$$

$$\text{paint} \vdash \mathsf{VP}_{(6,7)\backslash \mathsf{NP},(3,5)}/\mathsf{NP}_{(3,5)} : \lambda x.paint'x$$

Recognition again

- Given SCGG $\mathcal{G}$ and string pair u, v:
  - Construct a CCG $\mathcal{G}'_L$ producing all and only the set of valency trees of derivations of u.
  - Project the nodes of the valency trees through the synchronous lexicon to obtain CCG $\mathcal{G}'_R$.
  - Parse v with $\mathcal{G}'_R$.

Why not arbitrary regular languages?

(abb)*

intersected with

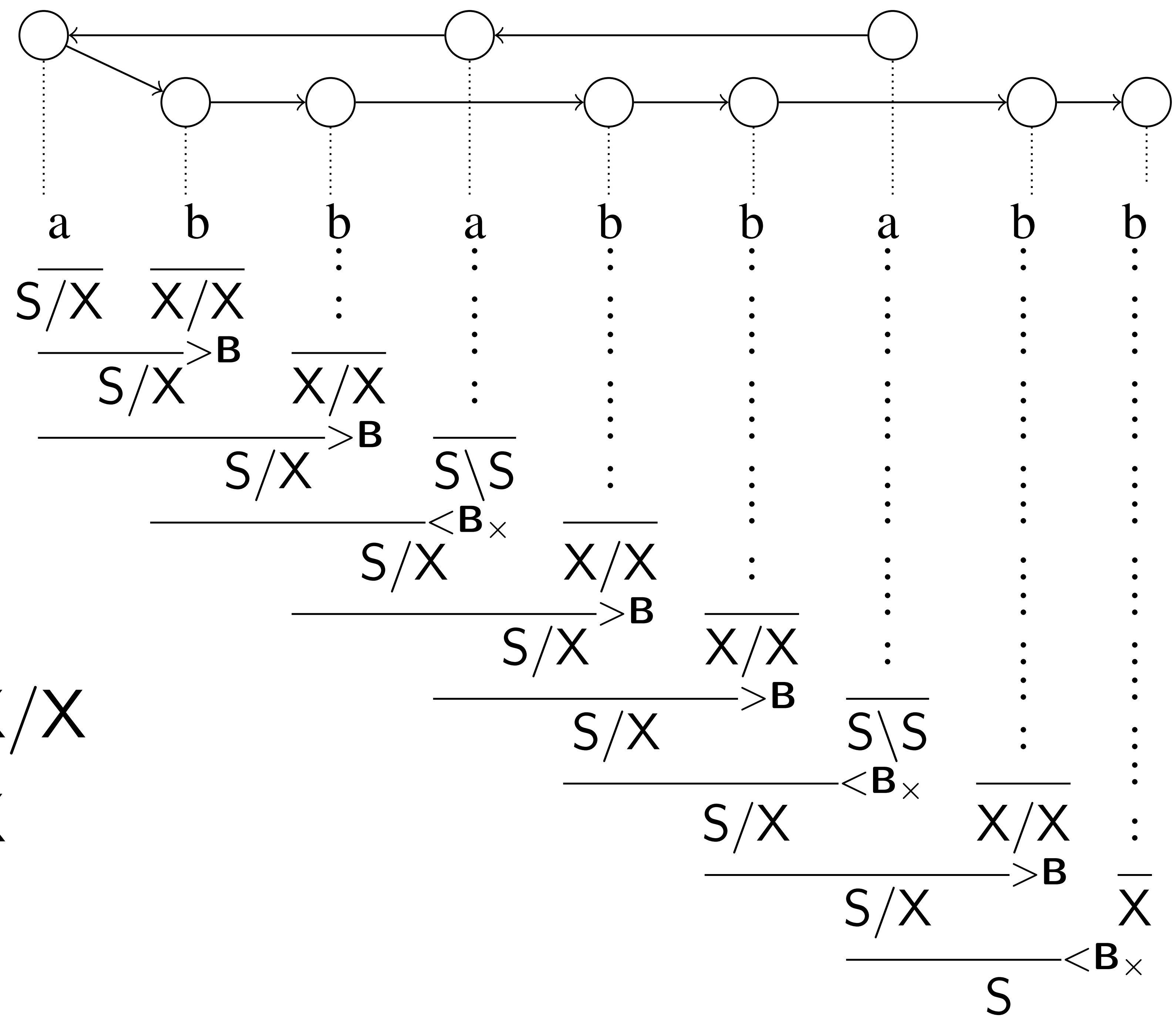
a $\vdash$ S\S   b $\vdash$ X/X

a $\vdash$ S/X   b $\vdash$ X

Why not arbitrary regular languages?

(abb)*

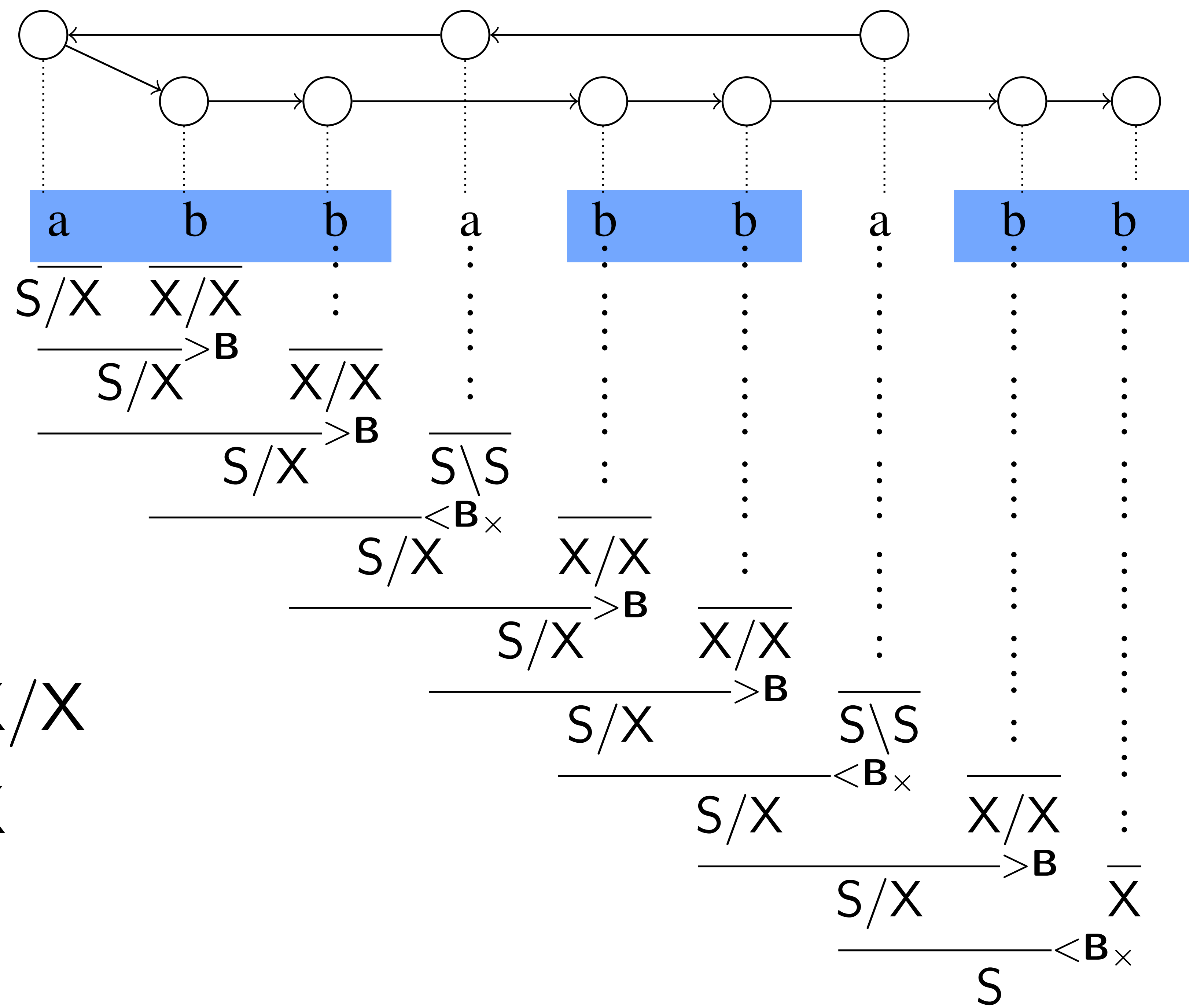
intersected with



a $\vdash$ S\S b $\vdash$X/X

a $\vdash$ S/X b $\vdash$X

Why not arbitrary regular languages?



(abb)*

intersected with

a ⊢ S\S   b ⊢X/X

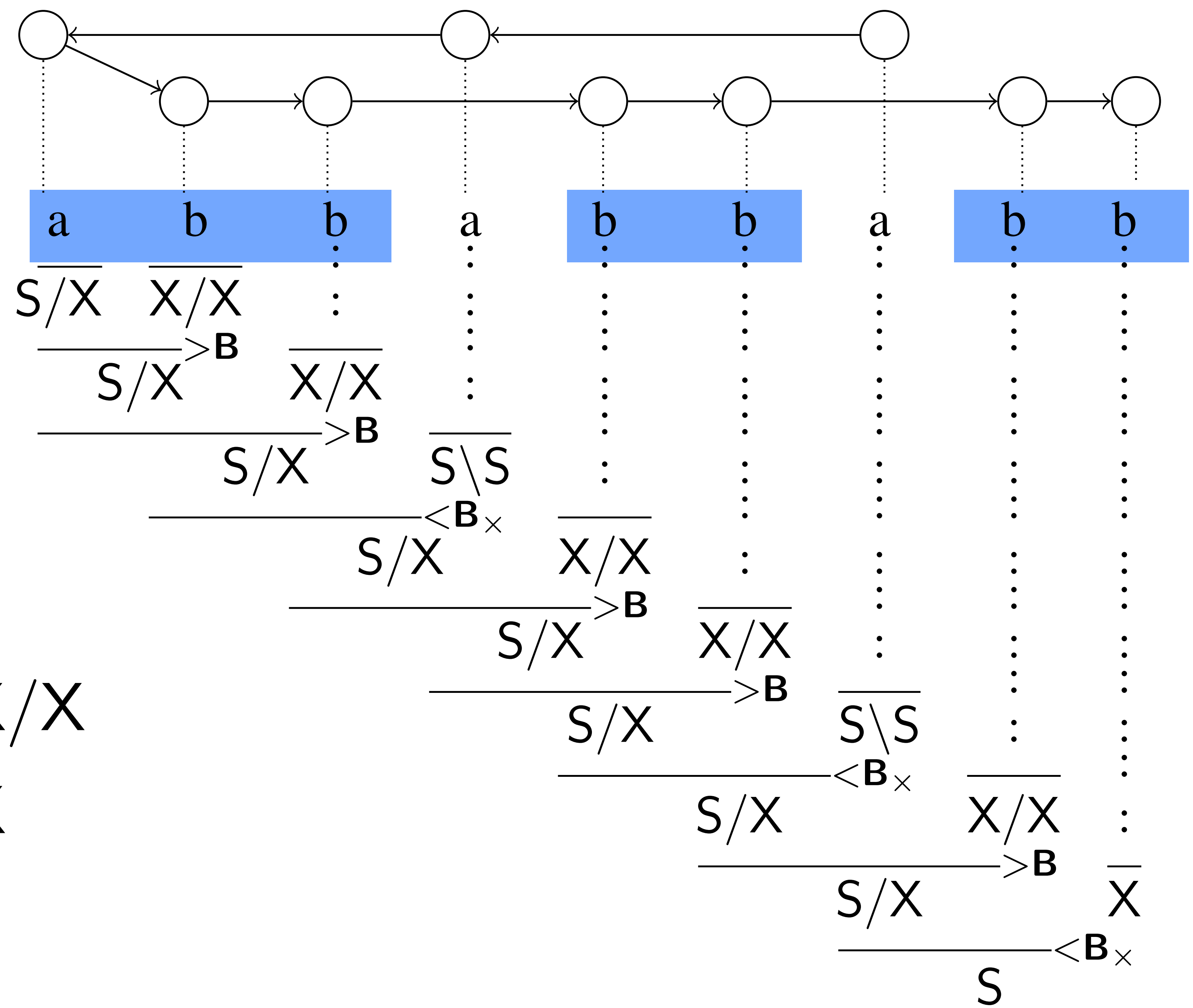
a ⊢ S/X   b ⊢X

Why not arbitrary regular languages?

Yield of valency tree rooted at leftmost *a*

(abb)*

intersected with

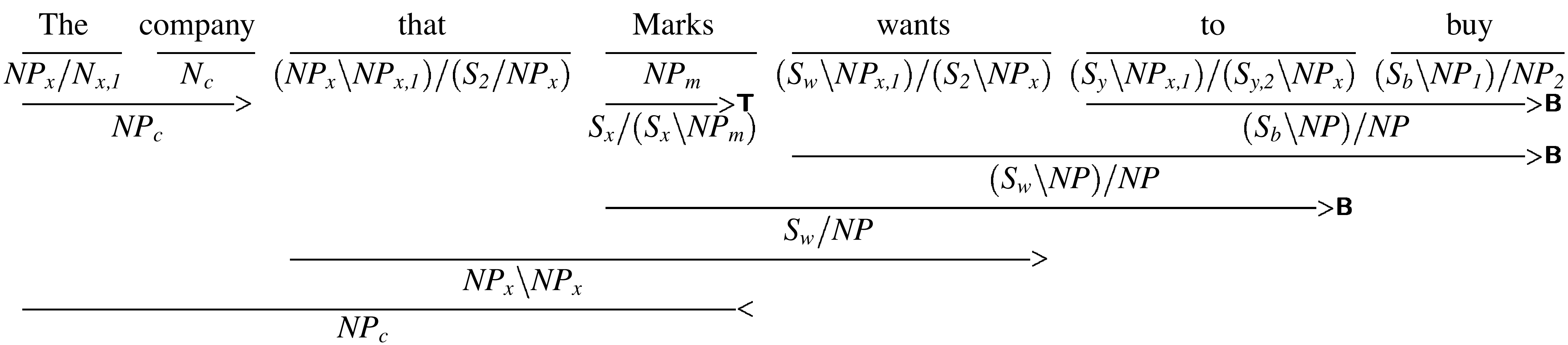


$\mathrm{a} \vdash S\backslash S$ $\mathrm{b} \vdash X/X$

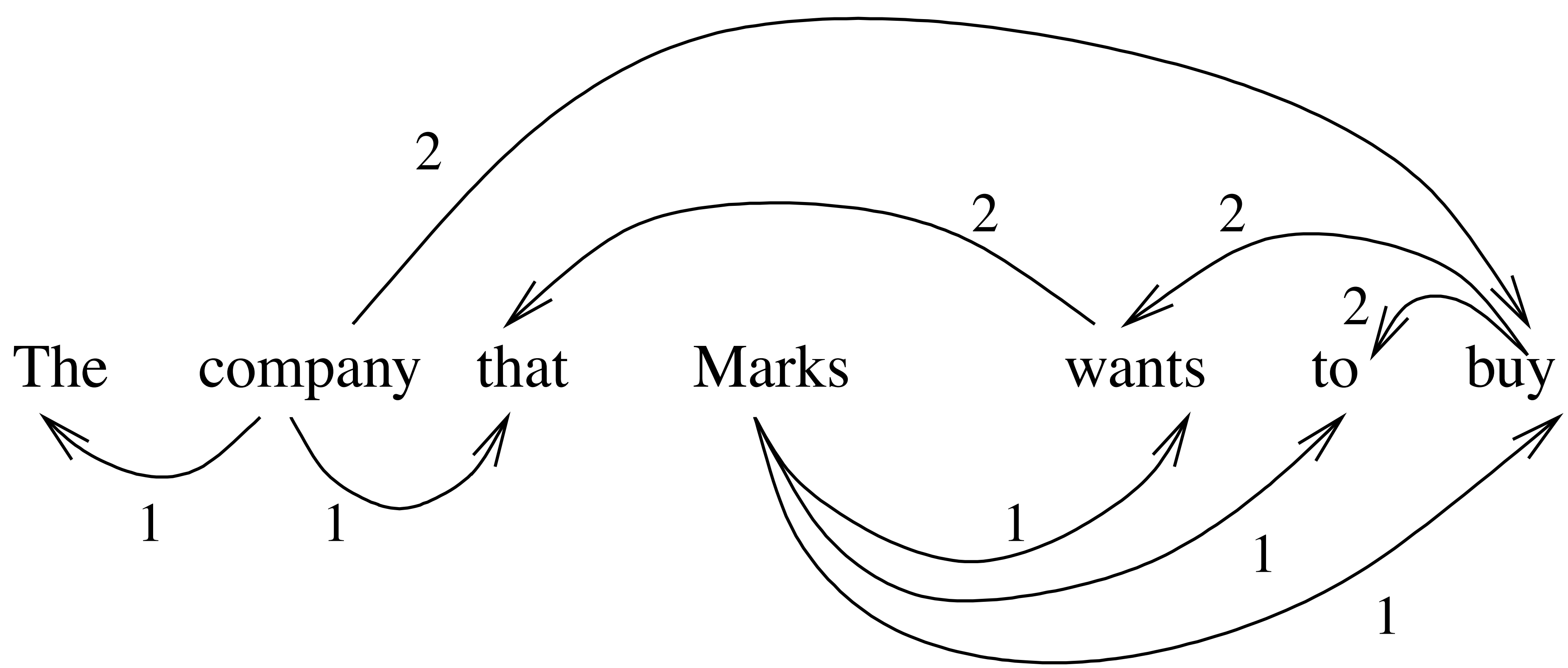
$\mathrm{a} \vdash S/X$ $\mathrm{b} \vdash X$

CCG and graph grammars

If bound variables appear more than once (Clark et al. 2002) ...



...Result is a dependency **graph**:



Open problems

- Higher-order categories: from Kuhlmann et al. 2010
- Type-raising: reverses a dependency edge.
- Substitution and the D combinator (Hoyt & Baldridge 2008) may permit Bar-Hillel-style construction.
- Non-pure CCG.
- Normal form yield expressions.
- CCG as LCFRS.

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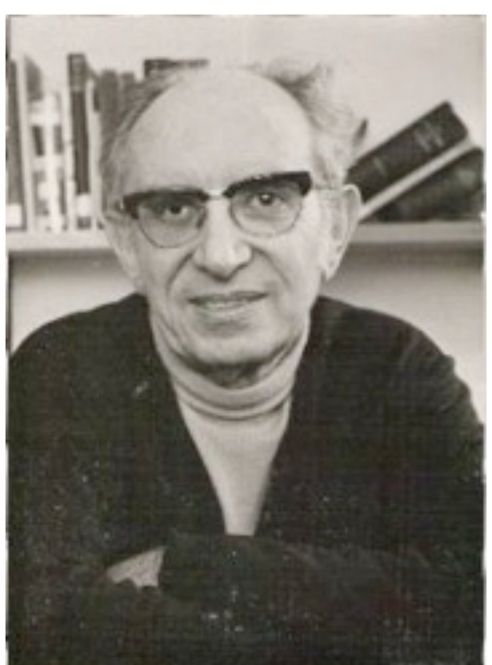
Yehoshua Bar-Hillel

... And remember your intersection constructions!