On counting wholes and parts: Cognitive and linguistic perspectives

Marcin Wągiel

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Introduction

Introduction

Counting

- common $\Rightarrow$ everyday experience
- cognitive $\sim$ linguistic perspectives
- three different though related concepts
  - count list $\Rightarrow$ recitation
  - arithmetic $\Rightarrow$ abstract operations
  - quantification $\Rightarrow$ cardinality of a set

(1) a. one, two, three, four, five, six,...
b. Three times two equals six.
c. three cats

Introduction

Outline

- Introduction
- Cognitive perspectives
- Linguistic perspectives
- Proposal
- Conclusions

Cognitive perspectives

Number sense

Two cognitive systems

Hyde (2011)

- OTS $\Rightarrow$ object tracking system
- ANS $\Rightarrow$ approximate number system

Figure 1: Object tracking

Figure 2: Approximate number

Number sense

Object tracking system

Carey (1998, 2009), Piazza (2010)

- mental ability to immediately enumarate small sets
- no counting via individuation
- manifests in infants

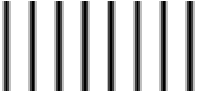


Figure 3: How many marks?

Number sense

Object tracking system

Carey (1998, 2009), Piazza (2010)

- mental ability to immediately enumarate small sets
- no counting via individuation
- manifests in infants

Figure 4: How many marks?

Number sense

Approximate number system

Feigenson et al. (2004), Nieder & Dehaene (2009), Cantlon et al. (2006)

- estimation of the magnitude of a collection
- no reliance on symbolic representation
- manifests in infants $\Rightarrow$ develops with age

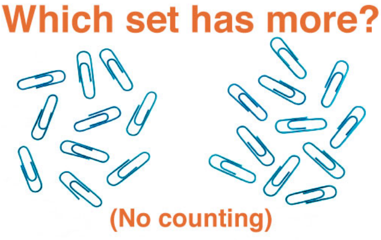


Figure 5: Compare

Number sense

Number sense in non-human animals

Davis & Pérusse (1998), Gallistel (1989), Dehaene (1997)

- primates $\Rightarrow$ operations on quantities
  - apprehension
  - comparison
  - approximate addition
- other mammals: dolphins, cats, rats
- also: birds, fish
- botanics $\Rightarrow$ plant arithmetic
- however, no evidence for symbolic addition except for the chimpanzee after long training

Psychology of counting

Implicit knowledge of counting in children

Gelman & Gallistel (1978)

- intuitive understanding of the cardinality of a set
- and its conservation under changes not affecting quantity
- each entity must be count once and once only
- 1 number cannot be associated with more than 1 entity
- no explicit formulation $\Rightarrow$ children are never taught that

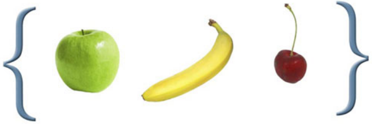


Figure 6: Enumerating sets

Psychology of counting

Innate principles of counting

Gelman & Gallistel (1978)

- stable order ⇒ ordered list of symbols
- 1-1 correspondence ⇒ symbols related to objects
- cardinality ⇒ determined by the last symbol

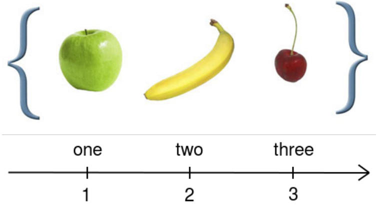


Figure 7: Counting and order

Psychology of counting

Acquisition of counting

Wynn (1990)

- children 6–18 months
  - stable order and 1-1 correspondence observed
  - fail when asked to give ‘two’ or ‘three’ objects
- 2,5 years
  - understanding that counting is an abstract procedure
  - applicable to different kinds of objects
- 3,5 years
  - order of recitation $\Rightarrow$ crucial
  - order of pointing at objects $\Rightarrow$ irrelevant
  - children indicate and correct subtle errors
- 4 years
  - counting can be generalized to novel situations

Psychology of counting

Quinean bootstrapping ⇒ crucial linguistic component
Carey (2009)

- learning the ordered list ⇒ relative order
- learning the meaning of symbols
- learning how the list represents number

(2) a. eeny, meeny, miny, mo,...
b. one, two, three, four, five, six,...

(3) $[\![\text{three}]\!] = 3$

Figure 8: Cardinality

Spatial integrity in counting

Object/substance distinction

Soja et al. (1991), Hauser & Carey (2003), Hauser & Spaulding (2006)

- innate ontological commitments
- manifested in infants
- assumptions ⇒ nature of objects
  - boundedness ⇒ natural boundaries
  - cohesion ⇒ parts stick together
  - movement across space along continuous paths
- substances ⇒ not expected to have those properties
- also in non-human animals

Spatial integrity in counting

Broken object experiments

Shipley & Shepperson (1990), Dehaene (1997), Melgoza et al. (2008)

- children between 3 and 4 years
- count only discrete integrated objects

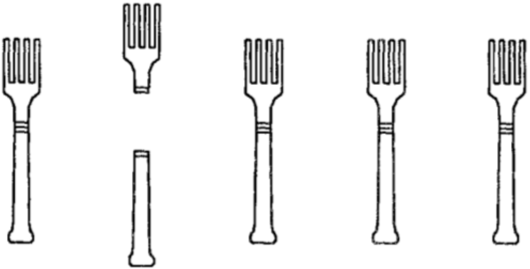


Figure 9: Relevance of integrity in counting

Spatial integrity in counting

Broken object experiments

Shipley & Shepperson (1990), Dehaene (1997), Melgoza et al. (2008)

- other forms of linguistic quantification
- comparative constructions and pluralization

Figure 10: Integrity in quantity comparison and pluralization

Part-whole structures

Ontological intuition

Varzi (2016), Priest (2014)

- Pre-Socratics ⇒ roots of mereology
  - entities ⇒ made up of smaller entities (parts)
- Plato ⇒ *Parmenides* and *Theaetetus*
  - unity ∼ arbitrary sum of parts
  - structure ⇒ arrangement of parts

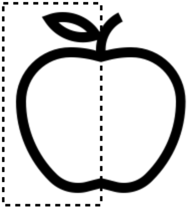


Figure 11: Material parthood

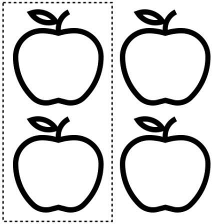


Figure 12: Individual parthood

Part-whole structures

Part-whole perception

Elkind et al. (1964), Kimchi (1993), Boisvert et al. (1999)

- simultaneous perception $\Rightarrow$ wholes $\sim$ collections of parts
- manifests in young children

Figure 13: Part-whole perception

Linguistic perspectives

Numeral phrases

OTS/ANS and grammar

Greenberg (1978), Hurford (1998, 2001), Rutkowski (2003)

- low vs. high numerals ⇒ different grammar
- high numerals ⇒ pattern with *many* in Slavic
- different case marking

(4) a. dvě / tři / čtyři kočky
two three four cat.NOM.PL

b. pět / mnoho koček
five many cat.GEN.PL — Czech

(5) a. dva / tri / četiri psa
two three four dog.GEN.SG

b. pet / mnogo pasa
five many dog.GEN.PL — BCS

Numeral phrases

OTS/ANS and grammar

Nelson & Toivonen (2000), Zabbal (2005), Ionin & Matushansky (2018)

- low vs. high numerals ⇒ different grammar
- different case marking in Finno-Ugric and Semitic

(6) a. kyeti / kulmâ poccuu
two three reindeer.ACC.SG

b. čiččâm / čyeti poccud
seven 100 reindeer.PART.SG — Inari Sámi

(7) a. ṯalāṯatu rijāl-i-n
three man-GEN-N

b. ṯalāṯūn rajul-a-n
thirty man-ACC-N — Standard Arabic

Numeral phrases

OTS/ANS and grammar

Aikhenvald (2000), Bale & Coon (2014)

- low vs. high numerals ⇒ different grammar
- classifier constructions

(8) a. sān běn shū
three CL book

b. *sān shū
three book — Mandarin

(9) a. na'n-ijig ji'nm-ug
five-AGR man-PL

b. asugom te's-ijig ji'nm-ug
six CL-AGR man-PL — Mi'gmaq

Counting/attributive numerals

Count lists across languages

Hurford (1998, 2001), Bylinina (2017), Wągiel & Caha (to appear)

- two sets of numerals in some languages ⇒ unexpected

(10) a. one, two, three,...

b. one cat, two cats, three cats,...

(11) a. raz, dva, tri,...
1 2 3

b. odin dom, dva doma, tri doma,...
1 house 2 houses 3 houses — Russian

(12) a. wieħed, tnejn, tlieta,...
1 2 3

b. ktieb wieħed, żewġ kotba, tlieta kotba,...
book 1 2 books 3 books — Maltese

Counting/attributive numerals

Count lists across languages

Hurford (1998, 2001), Bylinina (2017), Caha & Wągiel (2019)

- two sets of numerals ⇒ cross-linguistically common
- no distinction in English

LANGUAGE	NUMBER	ATTRIBUTIVE	COUNTING
German	2	zwei	zwo
Maltese	2	żewg	tnejn
Chinese	2	liǎng	èr
Hungarian	2	két	kettö
Basque	2	bi	biga

Counting/attributive numerals

Count lists across languages

Bylinina, Izard & Wągiel (in progress)

- bootstrapping theory predictions
  - faze when children use only counting numerals
  - children with 2 sets of numerals ⇒ slower acquisition
- linguistic/cognition interface
- ongoing project to test the predictions
- linguistic/cognition interface

(13) a. raz dom, dva doma, tri doma,...
1 house 2 houses 3 houses — Russian

b. ktieb wieħed, tnejn kotba, tlieta kotba,...
book 1 2 books 3 books — Maltese

Mass/count distinction

Countability ⇒ mass nouns ∼ count nouns
Jespersen (1913) among many others

- uncountable ∼ countable nouns
- grammatical category
- pluralization, compatibility with numerals
- intuition ⇒ object/substance distinction

(14) a. cat
b. cats
c. two cats

(15) a. mud
b. *muds
c. *two mud/muds

Mass/count distinction

Object mass nouns

Barner & Snedeker (2005), Chierchia (2010), Landman (2011)

- grammatical category ⇒ mass nouns
- denote discrete objects
- clash ⇒ grammar ∼ perception

(16) a. furniture
b. silverware
c. footwear

(17) a. nábytek
b. bižuterie
c. obuv — Czech

Mass/count distinction

Object mass nouns

Barner & Snedeker (2005), Chierchia (2010), Landman (2011)

- quantity comparison task
- object mass nouns pattern with count nouns
- attested in several typologically distinct languages

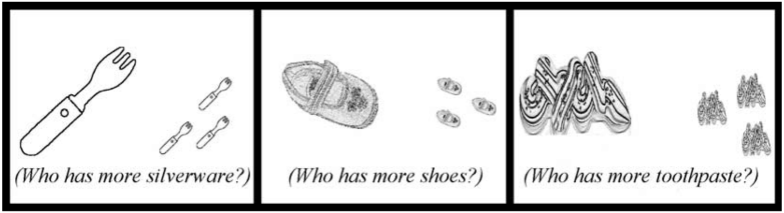


Figure 14: Object mass – count – mass

Proportional quantifiers

Individuation of parts

Krecz (1986), Markosian (1998), Acquaviva (2008)

- arbitrary portions ∼ structured parts
- spatial integrity
- cognitive salience
- structural or functional relevance
- natural language is sensitive to the distnction
- Czech lexicon: *část* ∼ *díl*

(18) a. A splinter is part of the table.
b. A leg is a part of the table.
c. #A splinter is a part of the table.

Proportional quantifiers

Individuation of parts

Krecz (1986), Markosian (1998), Acquaviva (2008)

- not all parts are spatially contiguous
- when countable ⇒ they need to be

(19) a. Dvě části kočky leží na silnici.
two parts cat.GEN lies on street.LOC
'Two parts of a cat lie on the street.'

b. Část koček leží na silnici.
part cats.GEN lies on street.LOC
'Some of the cats lie on the street.'

c. Dvě části koček leží na silnici.
two parts cats.GEN lies on street.LOC
'Two parts of cats lie on the street.' Czech

Proportional quantifiers

Contiguous vs. discontinuous parts

Wągiel (2018)

- natural language is sensitive to the distinction
- different structures ⇒ similar semantic effect
- diagnostics ⇒ the flag test

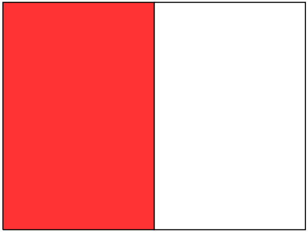


Figure 15: Flag AB

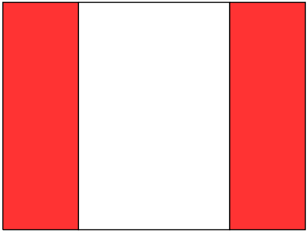


Figure 16: Flag ABA

Proportional quantifiers

Contiguous parts across languages

Wągiel (2018)

- dedicated syntactic construction

(20) a. Half the flag is red.
  (i) AB
  (ii) ABA
 b. A half of the flag is red.
  (i) AB
  (ii) #ABA

Proportional quantifiers

Contiguous parts across languages

Wągiel (2018)

- dedicated syntactic construction

(21) a. Guó qí de yí-bàn shì hóng de.
national flag DE one-half COP red DE
'Half the national flag is red.'
(i) AB
(ii) ABA

b. Bàn-miàn guó qí shì hóng de.
half-CL national flag COP red DE
'A half of the national flag is red.'
(i) AB
(ii) #ABA

Mandarin

Proportional quantifiers

Contiguous parts across languages

Wągiel (2018)

- dedicated morphological marker

(22) a. Połowa flagi jest czerwona.
half flag.GEN is red
'Half the flag is red.'
(i) AB
(ii) ABA

b. Połówka flagi jest czerwona.
half flag.GEN is red
'A half of the flag is red.'
(i) AB
(ii) #ABA

Polish

Proportional quantifiers

Contiguous parts across languages

Wągiel (2018)

- dedicated morphological marker

(23) a. Die Hälfte von der Fahne ist rot.
the half of the flag is red
'Half the flag is red.'
(i) AB
(ii) ABA

b. Die eine Hälfte der Fahne ist rot.
the a/one half the.GEN flag is red
'A half of the flag is red.'
(i) AB
(ii) #ABA

German

Proportional quantifiers

Contiguous parts across languages
Wągiel (2018)

- dedicated lexical item

(24) a. Metade da bandeira é vermelha.
half the flag is red
'Half the flag is red.'
(i) AB
(ii) ABA

b. Meia bandeira é vermelha
half flag is red
'A half of the flag is red.'
(i) AB
(ii) #ABA

Portuguese

Proportional quantifiers

Contiguous parts across languages

Wągiel (2018)

- different syntax

(25) a. De helft van de vlag is rood.
the half of the flag is red
'Half the flag is red.'
(i) AB
(ii) ABA

b. De halve vlag is rood.
the half flag is red
'The half of the flag is red.'
(i) AB
(ii) #ABA

Dutch

Counting and measuring

Counting and measuring are independent operations
Rothstein (2017), Wągiel (2018)

- distinct syntax and semantics
- counting indicates integrity $\Rightarrow$ measuring does not

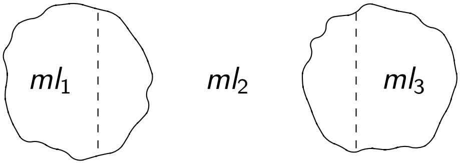


Figure 17: Inegrity in measuring and counting

(26) a. There are three mililiters of liquid on the table.
b. #There are three objects on the table.

Counting and measuring

Measuring is not sensitive to integrity

Wągiel (2018)

- numeral phrases ⇒ counting/measuring ambiguity
- counting ⇒ measuring shift
- possible but restricted

(27) CONTEXT: John is cooking with his child. They put three whole apples on a table. John says:

a. There are three apples on the table...

b. Let's count them together: one, two, three.

(28) CONTEXT: John is cooking with his child. They sliced three apples and put the slices into a bowl. John says:

a. There are three apples in the bowl...

b. #Let's count them together: one, two, three.

Interim summary

Cognitive perspective

- number sense ⇒ two different cognitive systems
- acquisition of counting ⇒ key linguistic component
- object/substance distinction ⇒ relevant
- part-whole structures ⇒ role of spatial integrity

Linguistic perspective

- low/high numerals ⇒ differ in grammar, not in meaning
- counting/attributive numerals ⇒ unexpected
- mass/count distinction ⇒ related to object/substance
- counting expressions ⇒ sensitive to integrity

Proposal

General counting principles

Counting ⇒ 1-to-1 correspondence with numbers

- non-overlap ⇒ disjoint entities (cf. Landman 2011, 2016)
- maximality ⇒ mereological exhaustivity
- integrity ⇒ individuated and integrated whole

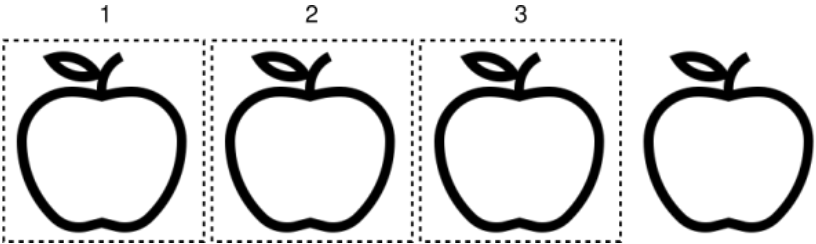


Figure 18: Counting

General counting principles

Illegal counting

- assigning a number to less than a whole entity
- summing up complementary parts
- overlapping entities

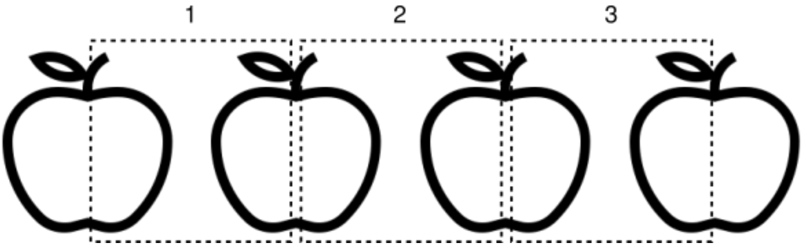


Figure 19: Illegal counting

General counting principles

Subatomic quantification

- counted parts $\Rightarrow$ maximal integrated entities
- counted parts cannot overlap

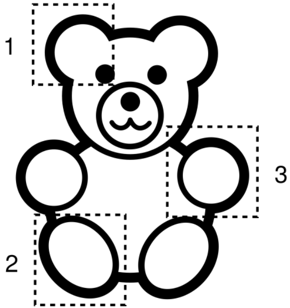


Figure 20: Counting of parts

General counting principles

Subatomic quantification

- counting discontinuous parts of an object
- overlapping parts

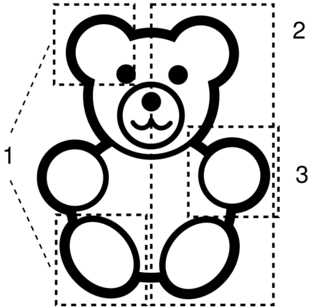


Figure 21: Illegal counting of parts

Formal implementation

Mereology

Leśniewski (1916), Leonard & Goodman (1940), Link (1983)

- parthood $\sqsubseteq$ and sum formation $\sqcup$
- entities equivalent to sums of their parts

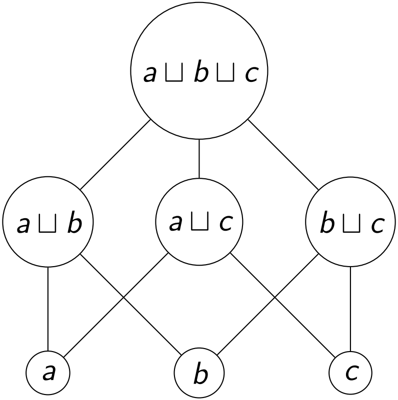


Figure 22: Semi-lattice

Formal implementation

Topology

Hausdorff (1914), Kuratowski (1922)

- spatial properties of space
- unaffected by continuous deformations of shape or size

Mereotopology $\Rightarrow$ mereology + topology

Whitehead (1920), Smith (1996), Casati & Varzi (1999), Varzi (2007)

- mereology augmented with topological relations
- no need for atomicity (having no proper parts)

Linguistic applications

Grimm (2012), Lima (2014), Henderson (2017), Wągiel (2018, 2019)

- mass/count distinction, collective/singulative number
- aggregates, swarms, Italian collective plurals, multipliers

Formal implementation

Mereotopology

Casati & Varzi (1999), Varzi (2007), Grimm (2012)

- connectedness C $\Rightarrow$ primitive relation
- reflexive, symmetric, not transitive, implied by overlap

(29) Parthood $\rightarrow$ connectedness
$\forall x \forall y[x \sqsubseteq y \rightarrow \forall z[\mathrm{C}(x,z) \rightarrow \mathrm{C}(z,y)]]$

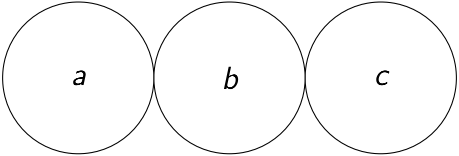


Figure 23: Connectedness and transitivity

Formal implementation

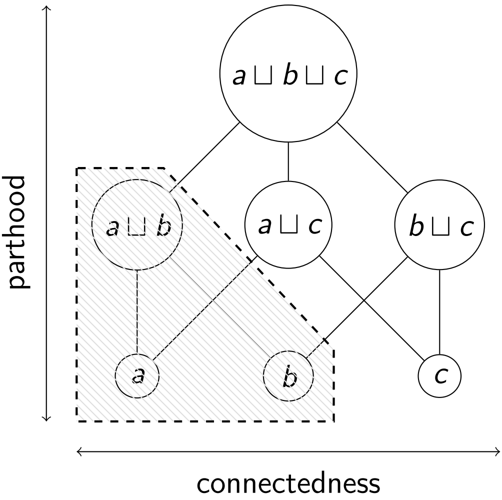


Figure 24: Parthood and connectedness

Formal implementation

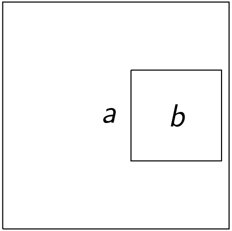


Figure 25: Internal part

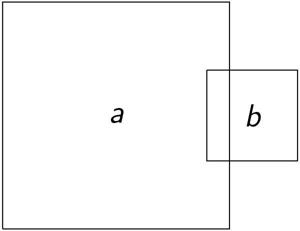


Figure 26: Internal overlap

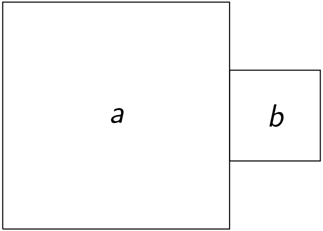


Figure 27: Tangential overlap

Formal implementation

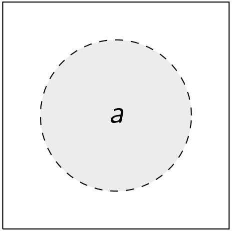


Figure 28: Interior

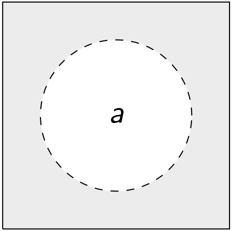


Figure 29: Exterior

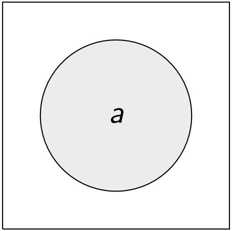


Figure 30: Closure

Formal implementation

Self-connected entity ⇒ cannot be divided into separated parts

(30) $\textsc{sc}(x) \stackrel{\text{def}}{=} \forall yz[\forall w(\textsc{o}(w,x) \leftrightarrow (\textsc{o}(w,y) \vee \textsc{o}(w,z))) \rightarrow \textsc{c}(y,z)]$

Strongly self-connected entity ⇒ entity's interior is $\textsc{sc}$

(31) $\textsc{ssc}(x) \stackrel{\text{def}}{=} \textsc{sc}(x) \wedge \textsc{sc}(ix)$

Maximally strongly self-connected relative to a property

(32) $\textsc{mssc}(P)(x) \stackrel{\text{def}}{=}$
$P(x) \wedge \textsc{ssc}(x) \wedge \forall y[P(y) \wedge \textsc{ssc}(y) \wedge \textsc{o}(y,x) \rightarrow y \sqsubseteq x]$

- strongly self-connected + maximality

Formal implementation

Capturing objects in mereotopology

- integrated wholes $\Rightarrow$ parthood and connectedness
  - entities that come in one piece
  - correspond to cognitive objects
- arbitrary sums $\Rightarrow$ only parthood
  - no topological notions involved

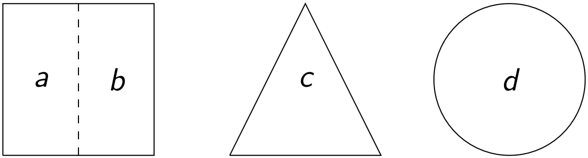


Figure 31: Wholes vs. sums

Formal implementation

Capturing counting

- count nouns $\Rightarrow$ denote integrated wholes
  - $\textsc{mssc}$ lexically encoded

(33) $[\![\text{apple}]\!] = \lambda x[\textsc{mssc}(\textsc{apple})(x)]$

- numerals $\Rightarrow$ require integrated wholes
  - root $\Rightarrow$ reference to a natural number
  - $\text{CL}_{\#}$ $\Rightarrow$ $\textsc{mssc}$ presupposition + measure function $\#(P)$

(34) $\forall P \forall x[\#(P)(x) = 1 \text{ iff } \textsc{mssc}(P)(x)]$

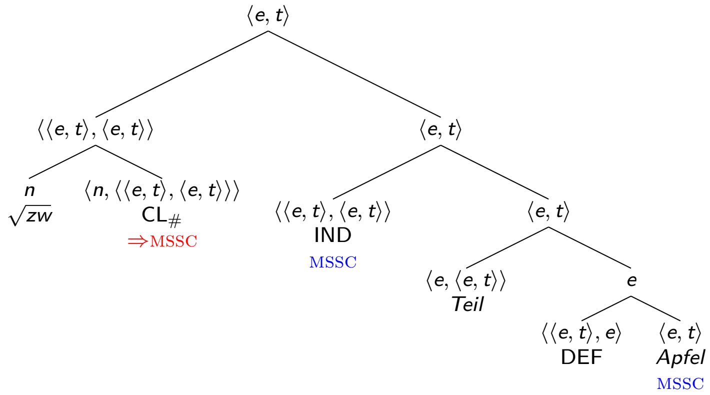
(35) $[\![\text{two}]\!] = [\![\text{CL}_{\#}]\!]([\![\sqrt{tw}]\!]) =$
$\lambda P : P_{\textsc{mssc}}\ \lambda x[{}^{*}P(x) \wedge \#(P)(x) = 2]$

Formal implementation

Counting in a partitive construction

(36) zwei Teile des Apfels
two parts the$_{\text{GEN}}$ apple$_{\text{GEN}}$
'two parts of the apple'

(37)



Formal implementation

Counting in a partitive construction

(38)

a. $[\![\text{Apfel}]\!] = \lambda x[\textsc{mssc}(\textsc{apple})(x)]$ MSSC

b. $[\![\text{DEF}]\!] = \lambda P[\textsc{max}(P)]$

c. $[\![\text{DEF Apfel}]\!] = \textsc{max}([\![\text{Apfel}]\!]) = \textsc{max}\big(\lambda x[\textsc{mssc}(\textsc{apple})(x)]\big)$

d. $[\![\text{Teil}]\!] = \lambda y \lambda x[x \sqsubset y]$

e. $[\![\text{Teil [DEF Apfel]}]\!] =$
$\lambda x[x \sqsubset [\![\text{DEF Apfel}]\!]] = \lambda x\Big[x \sqsubset \textsc{max}\Big(\lambda y[\textsc{mssc}(\textsc{apple})(y)]\Big)\Big]$

f. $[\![\text{IND}]\!] = \lambda P \lambda x[\textsc{mssc}(\pi(P))(x)]$ MSSC

g. $[\![\text{IND [Teil [DEF Apfel]]}]\!] = \lambda x[\textsc{mssc}(\pi([\![\text{Teil [DEF Apfel]}]\!]))(x)] =$
$\lambda x[\textsc{mssc}(\pi(\lambda z[z \sqsubset \textsc{max}(\lambda y[\textsc{mssc}(\textsc{apple})(y)])(z)]))(x)]$

h. $[\![\sqrt{\text{zw}}]\!] = 2$

i. $[\![\text{CL}_{\#}]\!] = \lambda n \lambda P : P_{\textsc{mssc}}\ \lambda x[{}^{*}P(x) \wedge \#(P)(x) = n]$ ⇒MSSC

j. $[\![\sqrt{\text{zw}}\ \text{CL}_{\#}]\!] = \lambda P : P_{\textsc{mssc}}\ \lambda x[{}^{*}P(x) \wedge \#(P)(x) = [\![\sqrt{zw}]\!]] =$
$\lambda P : P_{\textsc{mssc}}\ \lambda x[{}^{*}P(x) \wedge \#(P)(x) = 2]$

k. $[\![[\sqrt{\text{zw}}\ \text{CL}_{\#}]\ \text{[IND [Teil [DEF Apfel]]]}]\!] =$
$\lambda x[{}^{*}[\![\text{IND [Teil [DEF Apfel]]}]\!](x) \wedge \#([\![\text{IND [Teil [DEF Apfel]]}]\!])(x) =$
$2] =$
$\lambda x[{}^{*}(\lambda w[\textsc{mssc}(\pi(\lambda z[z \sqsubset \textsc{max}(\lambda y[\textsc{mssc}(\textsc{apple})(y)])(z)]))](w))(x) \wedge$
$\#(\lambda w[\textsc{mssc}(\pi(\lambda z[z \sqsubset \textsc{max}(\lambda y[\textsc{mssc}(\textsc{apple})(y)])(z)]))](w))(x) = 2]$

Conclusion

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Cognitive aspects of counting

- two independent cognitive systems ⇒ unified result
- 1-1 correspondence ⇒ entities and numbers

Linguistic aspects of counting

- natural language ⇒ sensitive to cognitive notions
- reflected in grammar

Quantification in natural language

- quantification over parts/wholes ⇒ identical restrictions
- counting ⇒ non-overlap, maximality and integrity

Conclusion THANKS!

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